Generalized tangent law for horizontal pairwise amplitude panning

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Abstract

Surround sound often relies on either simple mathematical models such as the tangent law or even more seriously, the perceptual localization curves due to level differences on horizontally arranged loudspeaker pairs at any head orientation. Several works exist showing for a lateral head orientation with regard to a stereophonic loudspeaker pair: (i) the amplitude in the back needs to be slightly enlarged to perceive the auditory event in the middle of the loudspeaker pair, (ii) the directional displacement of the auditory event from this position due to a small level difference change is larger than it would be for a frontal loudspeaker pair. This is what one can see from curves obtained by, e.g., Theile and Plenge, Pulkki, or Simon, Russel, and Rumsey. Nevertheless, a model of the localization for Simon's experiments consisting of a comprehensive set of head orientations cannot easily be validated using Theile's or Pulkki's localization curves. This is mainly due to the different angular loudspeaker spacings.

For a uniform set of perceptual data, this contribution presents a comprehensive experimental study. This is done to provide the relevant parameters of a generalized tangent law, based on perceptual localization curves for the loudspeaker pair spacings 30°, 45°, and 60°, with head orientations varied in 24 steps of 15°.

Introduction

In the 1960ies, several researchers investigated the direction of auditory events (*phantom sources*) in a great detail, even in third octaves, often only for frontal stereophonic pairs [1, 2, 3] and came up with models thereof, e.g. the tangent law by Clark and Dutton [4]. In the 1970ies, investigations were done on quadraphony and non-frontal loudspeaker pairs [5, 6, 7, 8]. However, the experimental results are not easy to compare, also not to newer studies [9, 10, 11]. The studies differ in what they tested: different louspeaker spacings, different head orientations.

Independently thereof, some models of multichannel reproduced auditory events localization can be found, e.g., in [2, 3, 12, 13], which, however, are frequency-dependent. By contrast, vector-base amplitude panning [14] is more practical as it implies a frequency- and direction-independent tangent law that holds for loudspeaker pairs or triplets set up at any direction and for broadband sounds. Recently, the theses of Frank [15] and Stitt [16] experimentally showed that the $\boldsymbol{r}_\mathrm{E}$ vector model is outperforming other predictions of broadband auditory events in typical playback conditions, despite its simplicity.

Anyway, some fundamental relationships for pairwise panning are not covered by the basic $\boldsymbol{r}_\mathrm{E}$ models: (i) By contrast to a frontal loudspeaker pair, the auditory event in the middle of a lateral loudspeaker pair requires slight non-zero level differences in dB, in favor of louder levels towards the side (or back) [8, 10, 11, 15, 17]. (ii) In lateral loudspeaker pairs, steep localization curves indicate substantially smaller level difference changes required to displace the auditory event from the middle of the loudspeaker pair [8, 11]. (iii) In a frontal setup with non-zero level difference, narrow-band sounds are offset farther from the middle of the loudspeaker pair at high center frequencies [1, 2, 3].

In this work, we focus on a closer investigation of (i,ii) in terms of a model and present an experimental study of pairwise amplitude-panned auditory events. The study goes further than the works of Theile [8] and Pulkki [10] for 60° loudspeaker pairs, or Simon [11] for 45° loudspeaker pairs, by enlarging the set of relative orientations of the head with regard to the loudspeaker pair, and by involving 30° loudspeaker pairs.

The main questions addressed by our study are:

- Which level difference is required for the phantom source in the middle between the loudspeaker pair for each head orientation?
- With which slope is such a phantom source displaced from the middle between the loudspeakers in $\frac{\tan\varphi}{\tan\alpha}\frac{40}{\ln 10}$/dB for each head orientation?

Ideally, targeted findings should be inter-subjective and involve any particular loudspeaker spacing and head orientation. The similarity of known perceptual localization curves to the tangent law [4] is utilized to simplify the experimental data acquisition. The experiment only needs to provide a few samples of the localizaton curve for each loudspeaker spacing and head orientation.

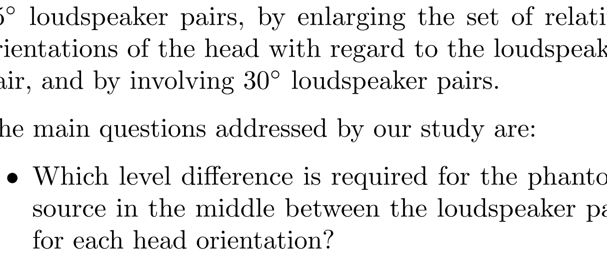

Figure 1: Modified tangent law with shift W and slope γ, with (γ, W) set to $(1, 0\text{ dB})$ for the dotted, $(2, 0\text{ dB})$ for the dashed, and $(3, 4\text{ dB})$ for the dash-dot curve.

Tangent law with slope and shift

The tangent law of stereophonic localization can be found formulated by Clark and Dutton in [4], depending on the gains g_1 and g_2 of two loudspeakers located at the angles $\pm\alpha$, and it defines an estimated perceived angle φ,

$$\frac{\tan\varphi}{\tan\alpha} = \frac{g_1 - g_2}{g_1 + g_2}. \tag{1}$$

To fit lateral localization curves more precisely, we propose a generalized tangent law. It is convenient to denote the gains depending on the level difference L in dB, $g_{12} = 10^{\pm\frac{L}{40}}$.

After substituting $10^{\pm\frac{L}{40}} = e^{\pm\frac{\ln 10}{40}L} = e^{\pm\sigma}$, we may simplify the fraction $\frac{g_1-g_2}{g_1+g_2} = \frac{e^{\sigma}-e^{-\sigma}}{e^{\sigma}+e^{-\sigma}} = \tanh(\sigma)$, hence the following re-formulation is valid

$$\tan\varphi = \tanh\sigma\ \tan\alpha. \tag{2}$$

Both functions of φ, σ are zero for a zero argument, $\tan(0) = \tanh(0) = 0$, and their slope is unity there $\tan'(0) = \tanh'(0) = 1$. A generalized stereophonic tangent law, formally of the kind $\tanh[\gamma(\sigma - \beta)]$ would allow a shift by W decibels and an adjustable slope γ for the center phantom source image, as shown in Fig. 1

$$\frac{\tan\varphi}{\tan\alpha} = \tanh\left[\frac{\ln 10}{40}\,\gamma\,(L - W)\right]. \tag{3}$$

Experimental setup

A setup of 24 Genelec 8020 in 15° degree steps at a radius of 1.5m at ear height was used to illuminate the introductory questions, see Fig. 2. The loudspeakers were set up in an anechoic chamber for the frequency range above 250 Hz. All loudspeakers were measured, level- and delay-compensated to the central listening position by a reference measurement microphone.

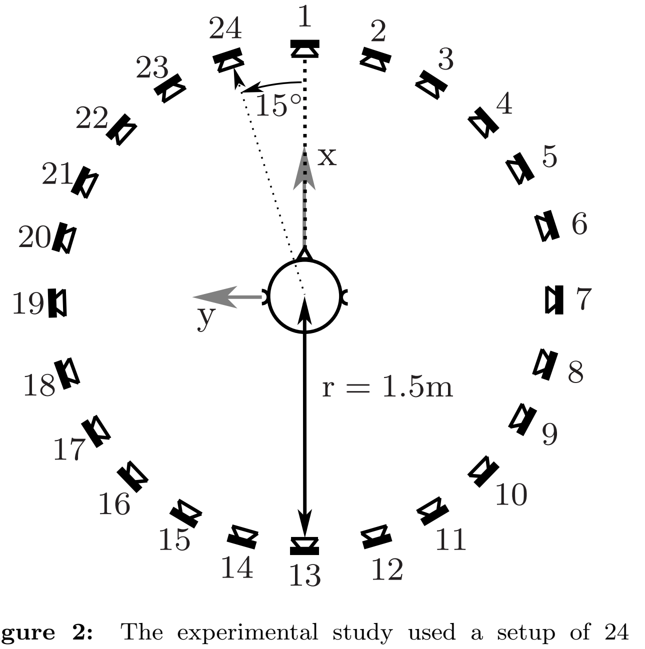

Figure 2: The experimental study used a setup of 24 loudspeakers on a ring. Any loudspeaker pair with 1, 2, or 3 loudspeakers (used as directional references) in between was selected to investigate pairwise panning for all orientations.

This setup allows to use any relative orientation of head with regard to loudspeaker pair in 15° angular steps. As loudspeaker pairs, loudspeaker spacings 30°, 45°, and 60° were used, as they are most relevant for amplitude panning and comparison to former studies.

For loudspeaker pairs of these spacings, the 15° equidistant loudspeaker setup always provides 1, 2, or 3 loudspeakers lying between the active pair, which are used to play back directional reference sounds in the experiment, as e.g. in [1, 10].

In the experiment, listeners matched the position of an amplitude-panned pink noise to the directional reference sound by moving either of two infinite MIDI pan pot controllers on their lap. Thereof, one was for coarse (1.5 dB) and the one for fine adjustment (0.5 dB).

The directional reference sound was a pink *complex tone* with a fundamental frequency of 40 Hz, harmonic components between 120 Hz and 20.4 kHz, each of which using a random phase offset,

$$r(t) = \sum_{k=3}^{510} \frac{\sin(2\pi\, 40\, k\, t + 2\pi\, \mathrm{rand}[k])}{\sqrt{k}}. \tag{4}$$

The reason to choose a different directional reference sound than pink noise was to avoid subjects getting distracted by close timbral similarity/dissimilarity in the directional matching task.

The envelope of both sounds was a sine-squared quarter-wave fade in and out of 200 samples length @44.1 kHz sampling rate (≈ 4.5 ms), with a 300 ms duration for each noise or complex tone burst. The periodically repeating sounds used a firing interval of 330 ms for the sequence $\langle$stimulus$\rangle$, $\langle$stimulus$\rangle$, $\langle$pause$\rangle$, $\langle$reference$\rangle$, $\langle$pause$\rangle$.

Subjects could confirm their adjustment and move to the next presentation by pushing a knob on the MIDI controller.

Conditions

The amplitude-panned pink noise should be adjusted to match (a) the 0° directional reference within 24 loudspeaker pairs of 30° aperture, (b) the ±7.5° reference directions within 24 loudspeaker pairs of 45° aperture, and (c) the ±15° and 0° reference directions within 24 loudspeaker pairs of 60° aperture. The total of $24 + 2 \times 24 + 3 \times 24 = 144$ adjustment tasks was given twice to each subject, each time in random overall order to avoid order effects, and listeners had a break in between. Listeners took about 70 min on average to complete the $144 + 144$ adjustment tasks.

After a short familiarization phase, subjects were instructed to look forward during the entire experiment and to adjust the center of the pink noise location to match the direction of the reference sound by using the coarse and fine knobs, and that they may use back-and-forth rotations of the knobs to achieve the matching goal.

The 5 listeners that took part in the experiments were experienced listeners in spatial audio in the age between 29 and 52. The authors were two of the subjects.

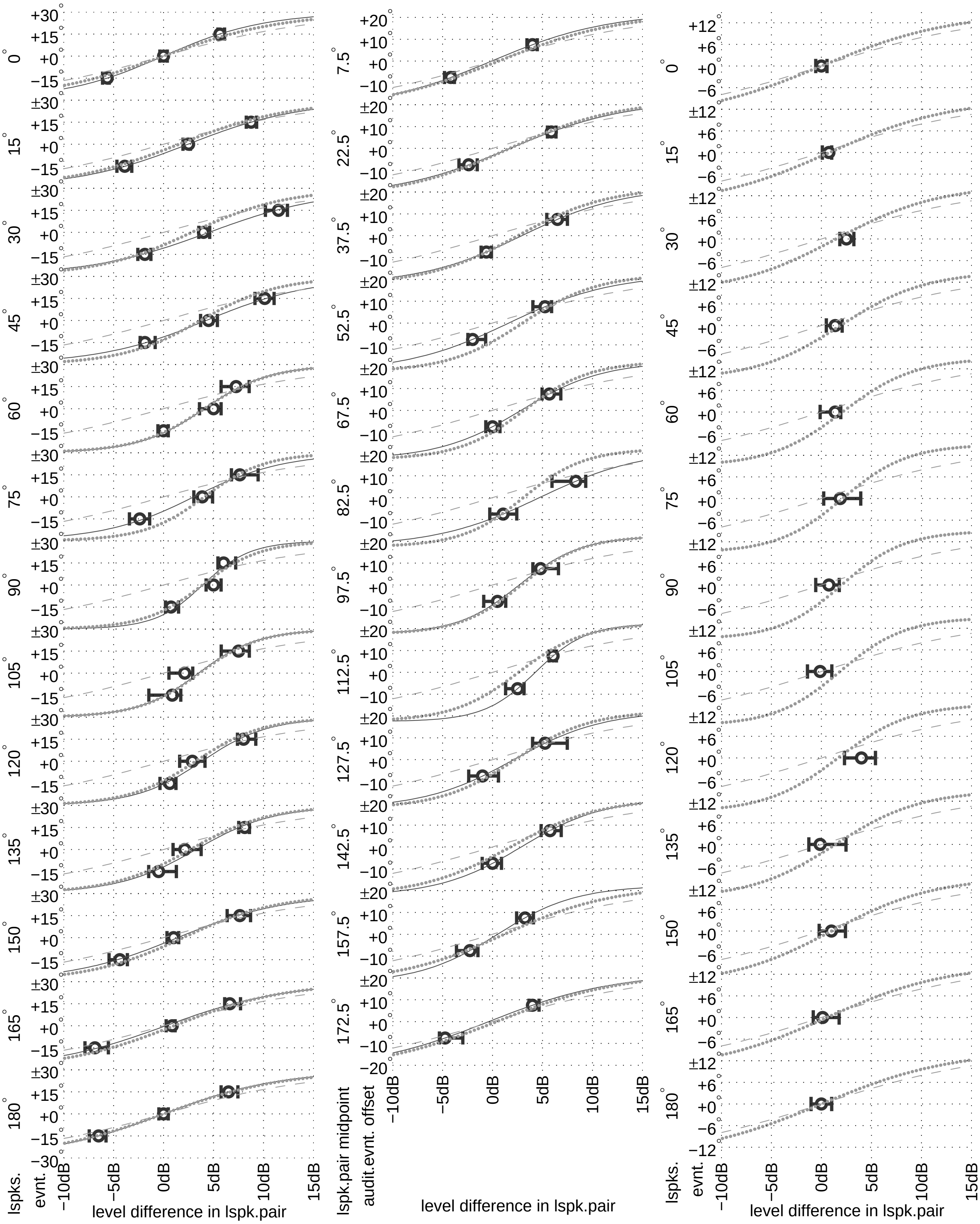

Figure 3: Listening test results for loudspeaker pairs of 60° spacing (left column), 45° spacing (middle column), and 30° spacing (right column). The horizontal axis is used to draw level differences found by experiment in each column. In each row, the vertical axis represents the desired offset of the auditory event with regard to the midpoint of the loudspeaker pair, which is varied by 15° steps in in each column. Experimentally found level differences are drawn as black ring markers (medians) and whiskers indicate the 95% confidence intervals. For reference, the dashed gray curve shows the tangent law, the generalized tangent law is shown fitted to the data of each loudspeaker pair is shown in gray (continuous) and as dotted curve after employing a model of $W_{2\alpha}(\theta)$ and $\gamma(\theta)$.

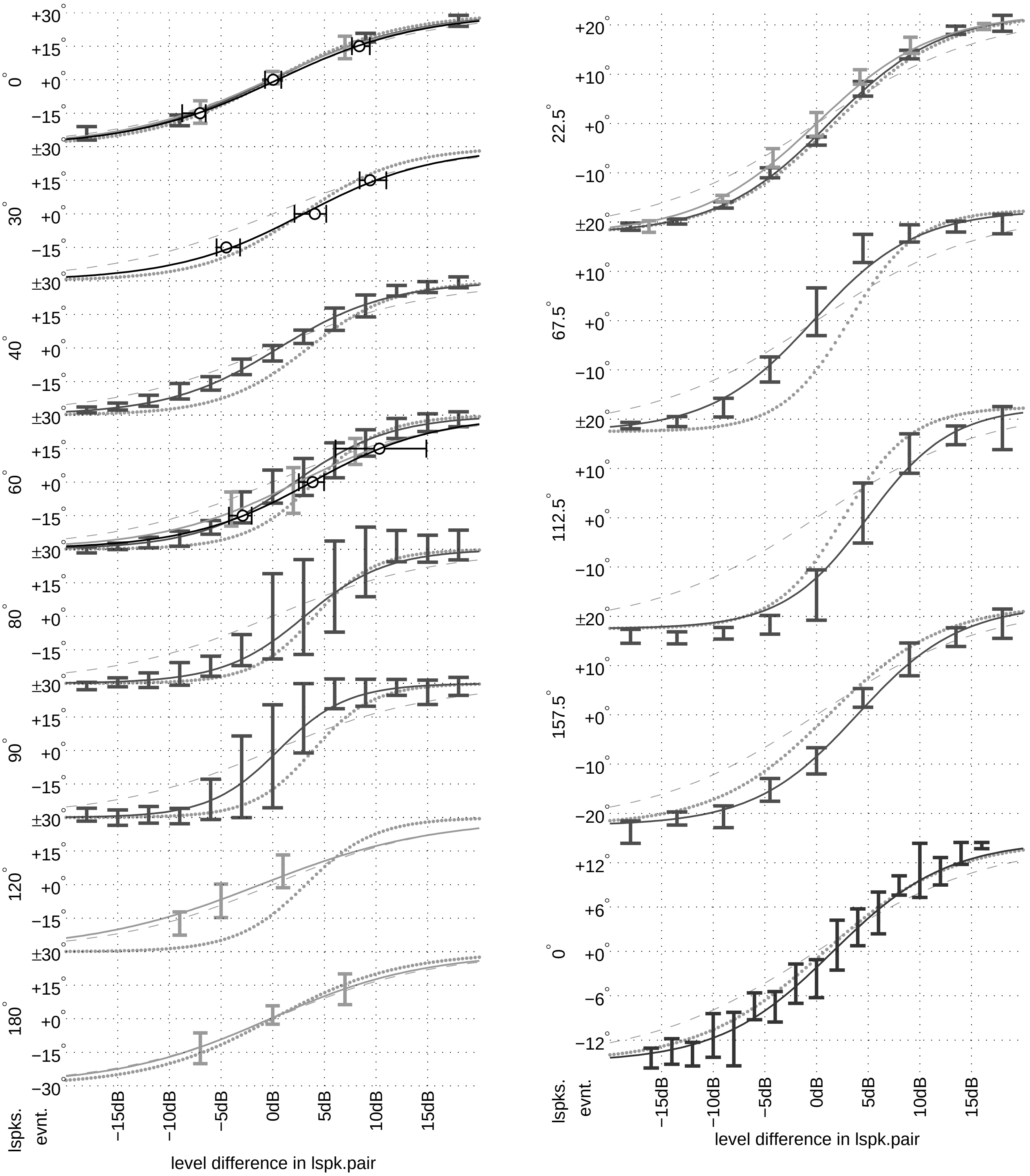

Figure 4: The heavy dotted curves display the generalized tangent law defined by the experiments in this study and the dashed gray curve is the classical tangent law. Both curves are shown in comparison with experimental data from literature, to each of which a tangent law curve is drawn with fitting slope and shift. The left column displays curves for the 60° loudspeaker spacing, with each row for a different loudspeaker pair midpoint, compared with curves from Theile and Plenge [8] of their first experimental set (dark gray, bold) and of their second one (light gray, bold); data from Pulkki's experiments [10] are shown as black thin lines and markers. The right column displays data for 45° loudspeaker spacing from Simon [11] in its upper 4 rows; light gray in the first row are from [15]. The last row on the right displays Martin's data [9] for a 30° segment of a 5.1 setup.

Results

Left-right asymmetries were removed from the experimental data. This was done by subtracting from all left-right opposing sides half the asymmetry-caused level difference. For each loudspeaker pair, the asymmetry was detected by the difference of the medians for all left-right-opposing panning directions, and the median thereof. Symmetrization largely used expressions below ±1.5 dB for most frontal and dorsal angles and allowed there being 20 responses for each of the symmetrized conditions. It allowed to neatly plot the level differences in terms of medians and confidence intervals for all conditions, using the midpoint angles between 0° and 180° to separate the loudspeaker pairs in Fig. 3.
Total standard deviations for level differences were 3.6 dB for 60°, 4.3 dB for 45°, and 4.9 dB for the 30° spacing. Thus variances are inversely proportional to loudspeaker spacings, hence appear angular-resolution-induced. The increase of slope to lateral directions and a shift towards the rear is found in the results, consistently with literature.

Fitting slope and shift

Given perceptual data for every individual loudspeaker pair of the experiment, we are able to retrieve the parameters W and γ of the generalized tangent law in Eq. (3). The least-square-error solutions for the tested 60°, 45° and 30° loudspeaker spacings are:

$$\gamma_{60^\circ} = \frac{2\,\mathrm{artanh}\frac{\tan 15^\circ}{\tan 30^\circ}}{L_+ - L_-}, \quad W_{60^\circ} = \frac{L_- + L_0 + L_+}{3}, \tag{5}$$

$$\gamma_{45^\circ} = \frac{\mathrm{artanh}\frac{\tan 7.5^\circ}{\tan 22.5^\circ}}{L_+ - L_-}, \quad W_{45^\circ} = \frac{L_- + L_+}{2}, \tag{6}$$

$$W_{30^\circ} = L_0. \tag{7}$$

For the 30° pair, with only the direction $\phi = 0^\circ$ tested, only the shift W_{30° can be detected.

Generalized tangent law

It is desirable to estimate the parameters W and γ for Eq. (3) in general. Hence, the attempt here is to model $W_{2\alpha}(\theta)$ and $\gamma_{2\alpha}(\theta)$ for any loudspeaker pair of a given spacing 2α and midpoint angle θ. It is reasonable to assume that the level differences for the midpoint auditory events depend on a directivity pattern $w_{\mathrm{dB}}(\theta)$, as in [10, 15, 17]. This assumption allows to postulate a model of $W_{2\alpha}(\theta)$ for any orientation θ and spacing 2α

$$W_{2\alpha}(\theta) = w_{\mathrm{dB}}(\theta+\alpha) - w_{\mathrm{dB}}(\theta-\alpha) \text{ in dB}. \tag{8}$$

We intend to find $w_{\mathrm{dB}}(\theta)$ in terms of coefficients of a 3rd order left-right symmetric cosine series

$$w_{\mathrm{dB}}(\theta) = \sum_{k=1}^{3} c_k \cos(k\theta).$$

An equation system needs to be solved seperately for all 2α-spaced loudspeaker pairs of varying midpoints θ_l, given each midpoint level difference

$$W_{2\alpha}(\theta_l) = \sum_{k=1}^{3} c_k \left\{ \cos[k(\theta_l+\alpha)] - \cos[k(\theta_l-\alpha)] \right\}. \tag{9}$$

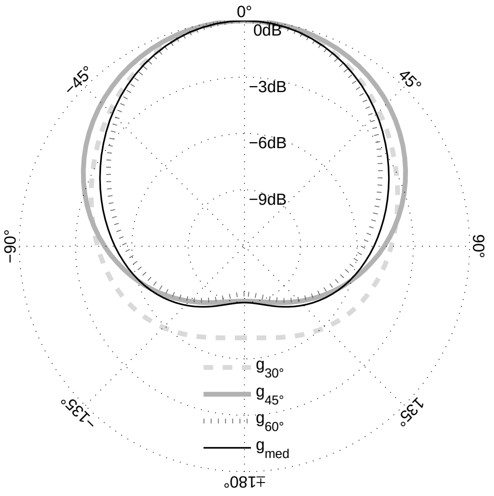

Figure 5: Directivity patterns obtained from shifts of the midpoint auditory event on any loudspeaker pair of 30° (dark gray, dotted), 45° (gray, continuous), 60° (light gray, dashed), and a median model (black, continuous) Eq (10).

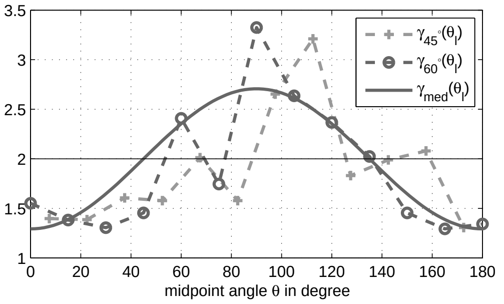

Figure 6: The slope of the panning curve γ varies with the loudspeaker midpoint angle θ, however, maintains its tendency across experimentally investigated loudspeaker spacings 2α. The model of Eq. (11) (continuous) fits both the datasets $\gamma_{60^\circ}(\theta_l)$ (dark gray, dashed) and $\gamma_{45^\circ}(\theta_l)$ (light gray, dashed).

By solving separately for all loudspeaker spacings of the experiments, and by re-expanding the median across all spacings α in the angular domain in terms of c_k, we obtain the following formulation

$$w_{\mathrm{dB}}(\theta) = -4.8 + 4.2\cos\theta + 0.3\cos 2\theta + 0.3\cos 3\theta, \tag{10}$$

in which the constant offset −4.8 dB was added for normalization, as shown in Fig. 5.
The slopes $\gamma_{60^\circ}(\theta_l)$ and $\gamma_{30^\circ}(\theta_l)$ are plotted in Fig. 6 where they are compared to a simple analytic curve

$$\gamma(\theta) = 2 - \tfrac{1}{\sqrt{2}}\cos(2\theta). \tag{11}$$

Using $w_{\mathrm{dB}}(\theta)$ Eq. (10) to get $W_{2\alpha}(\theta)$ from Eq. (8), and $\gamma(\theta)$ Eq. (11), we can write a closed form expression of an overall generalized tangent law

$$\frac{\tan\varphi}{\tan\alpha} = \tanh \frac{\gamma(\theta)\,[L - W_{2\alpha}(\theta)]\ln 10}{40}, \tag{12}$$

or using θ_1 and θ_2 for the loudspeaker angles, keeping θ for the midpoint, $g_{1,2}$ for their gains, and $w(\theta) = 10^{\frac{w_{\mathrm{dB}}(\theta)}{20}}$,

$$\frac{\tan\varphi}{\tan\alpha} = \frac{[g_1\,w(\theta_1)]^{\gamma(\theta)} - [g_2\,w(\theta_2)]^{\gamma(\theta)}}{[g_1\,w(\theta_1)]^{\gamma(\theta)} + [g_2\,w(\theta_2)]^{\gamma(\theta)}}. \tag{13}$$

Comparison to literature

Experimental data from several sources in literature are compared with the overall generalized equation from Eq. (12) in Fig. 4. The match is better than for the simple tangent law curve in most cases, however, the match is not as gapless as with the comprehensive data provided here. One can speculate that main reasons might be methodological ones in some papers. A different geometrical bias could emerge, e.g., from drawn indication of the perceived angle.

What would easily adapt the model to fit the literature better is a modified shift W, see curves for 40°, 60°, 80°, 90°, 120° in the left column and for 67.5°, 112.5°, 157.5° in the right column. Directivity-induced level differences from literature appear to be over-estimated by our model for lateral directions from $40° \dots 90°$ in both columns. However, in the dorsal lateral directions, the indication is contradictory: the 120° curve of the left column indicates over-estimation by -3 dB, whereas the 112.5°, 157.5° curves indicate under-estimation of W by $+2$ dB.

Conclusion and outlook

We presented a generalized tangent law for pairwise amplitude panning using horizontal loudspeaker pairs of any spacing at any head orientation. It utilizes an underlying directivity as in [10, 15, 17] to characterize the level difference required to evoke auditory events in the midpoint of lateral loudspeaker pairs. Moreover, a slope parameter reflects the fact that displacement is achieved with smaller level difference changes on lateral loudspeaker pairs. Both parameters were fit to the experimental data and gathered in a new tangent law that incorporates shift and slope.

Comprehensive experimental data for model calibration was acquired using loudspeaker pairs spaced by 30°, 45° and 60° selected from a ring of 24 loudspeakers. Level differences were adjusted to evoke auditory events matching all reference directions enclosed by the loudspeaker pair. The dataset contains level differences for the 144 conditions.

It might be interesting to model the shift W by measured head-related transfer functions in the future, as in [15].

The question of in how far the reporting method of the experiment influences responses for lateral panning angles was not covered in this study, nor could we investigate the frequency-proportional slope for frontal panning, which is supported, e.g., by the experiments of Mertens [2] and Wendt [3].

The $\boldsymbol{r}_\mathrm{E}$ vector model has recently turned out to be a simple practical model of phantom source localization with good precision for amplitude-panned broadband sounds. Future work might involve the refinement of the generalized tangent law to multiple loudspeakers. It is possible that an equation of the form

$$\boldsymbol{r}_\gamma^w = \frac{\sum_l \theta_l \, (g_l \, w(\theta_l))^{\gamma(\boldsymbol{r}_\mathrm{E}/\|\boldsymbol{r}_\mathrm{E}\|)}}{\sum_l (g_l \, w(\theta_l))^{\gamma(\boldsymbol{r}_\mathrm{E}/\|\boldsymbol{r}_\mathrm{E}\|)}}$$

might be successful.

Acknowledgments

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