



Gradientenmikrofone höherer Ordnung

Franz Zotter

Gastvorlesung 25.1.2012

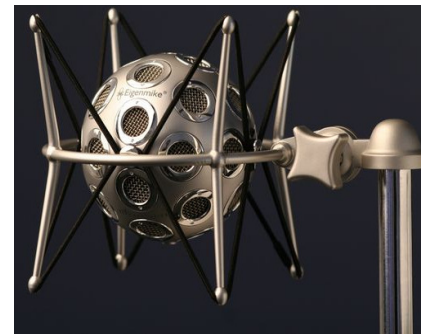
ITA RWTH Aachen

Elektroakustik / akustische Messtechnik



Ausblick

- frequenzunabhängige akustische Gradientenbildung begrenzt
- akustischen Lösungen (Patente) existieren, Klangeigenschaften?
- Übersicht elektronische Gradientenbildung von Druckmikrofonen
- Frequenzunabhängige Richtwirkung in eine Richtung + Entwurf
- ...mit variabler Ausrichtung
- akustische Überprüfung durch Messung
- offene Fragestellungen

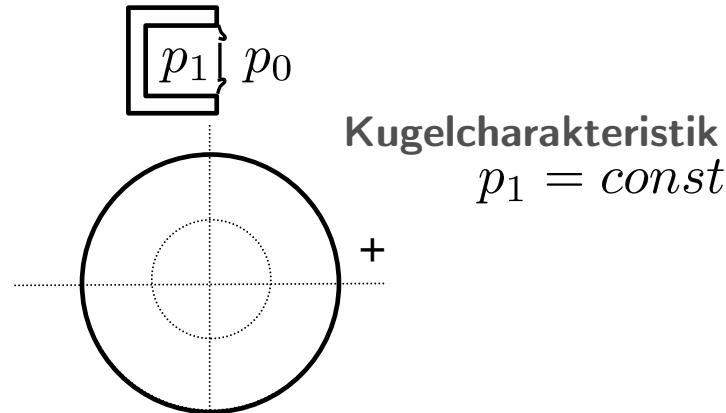


Verbreitete Mikrofontechnologie

(vgl. Technische Akustik, Lerch, Sessler, ?)

Druckdifferenz zwischen vor- und hinter einer Membran $p_0 - p_1$

- Druckempfänger



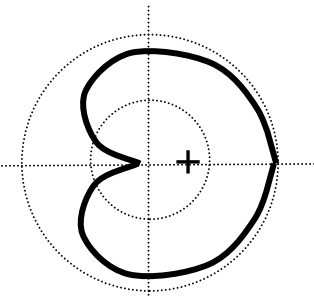
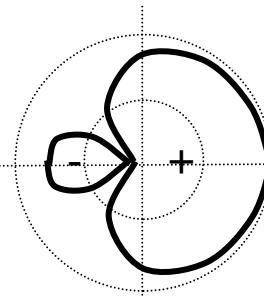
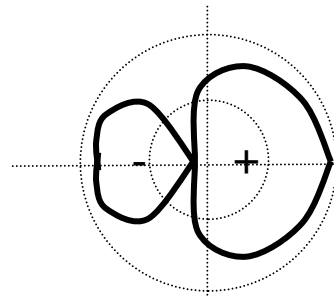
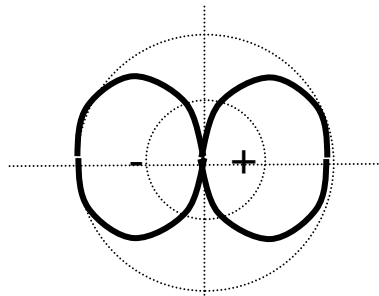
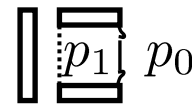
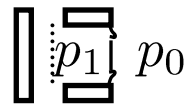
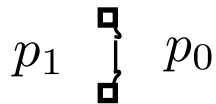
- Gradientenwandler mit Laufzeitglied o/m Luftwiderstandsglied

„Achter“-Charakteristik

„Hyperniere“

„Superniere“

„Nieren“-Charakteristik

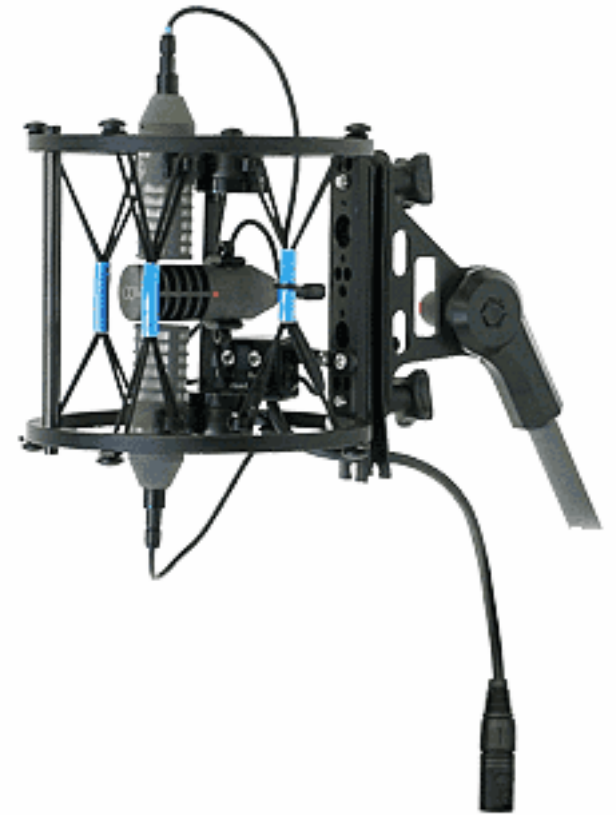
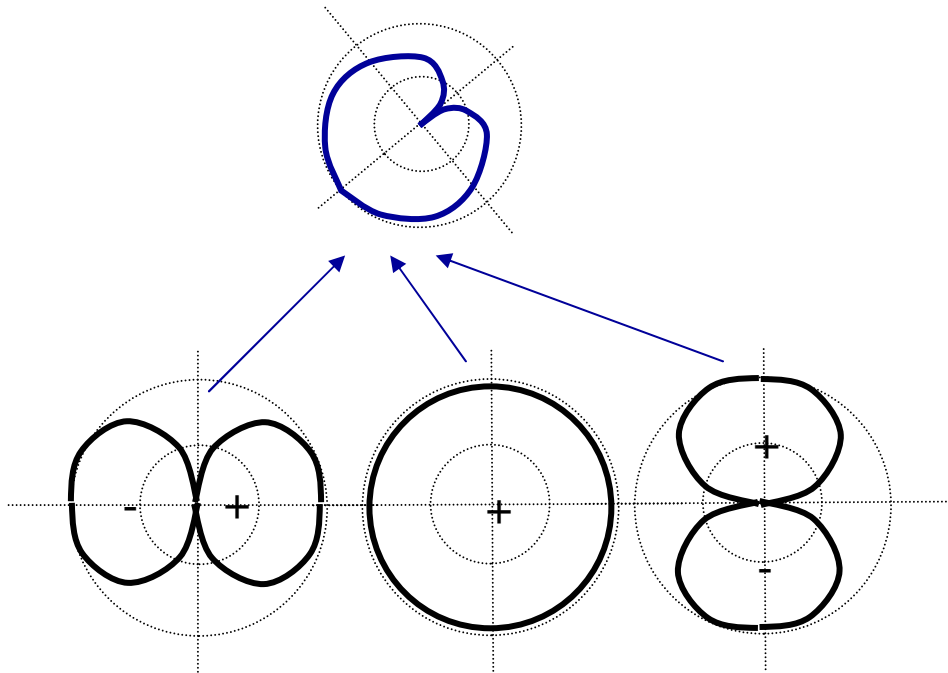


Akustische Einstellung über Dimensionierung Laufzeitglied und Luftwiderstand



Kombination von Richtwirkungen erster Ordnung

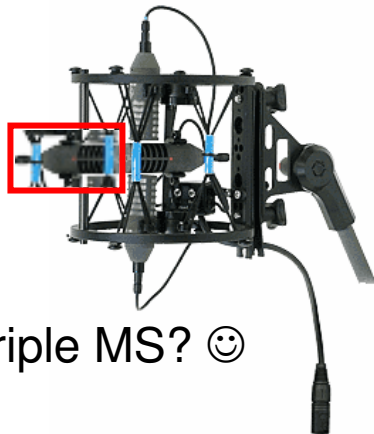
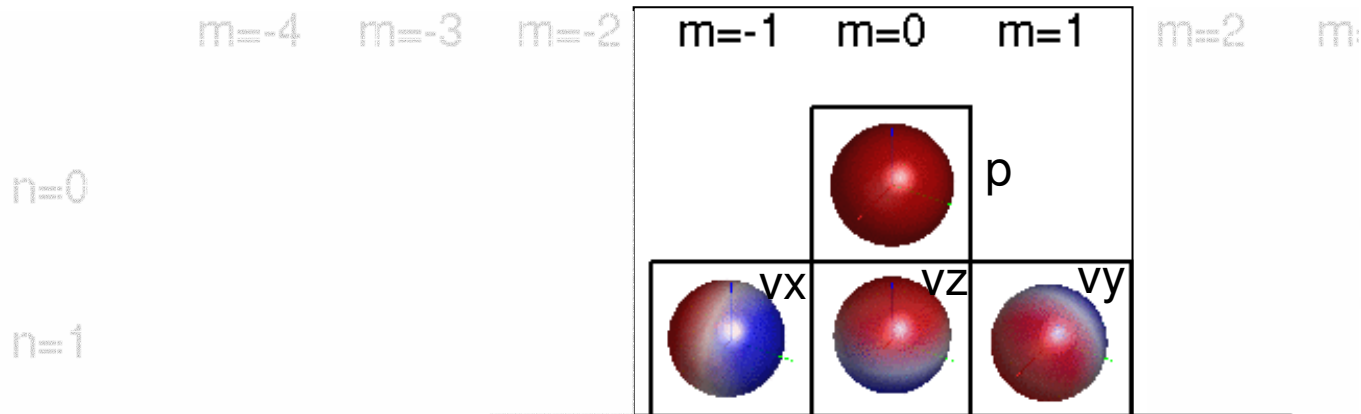
Erlaubt freie Ausrichtung und Bündelungseinstellung



Schöps, Doppel-MS

Räumlich kombinierte Richtwirkungen erster Ordnung

Kombination 4 untersch. ausgerichteter Nierenmikrofone



Triple MS? 😊

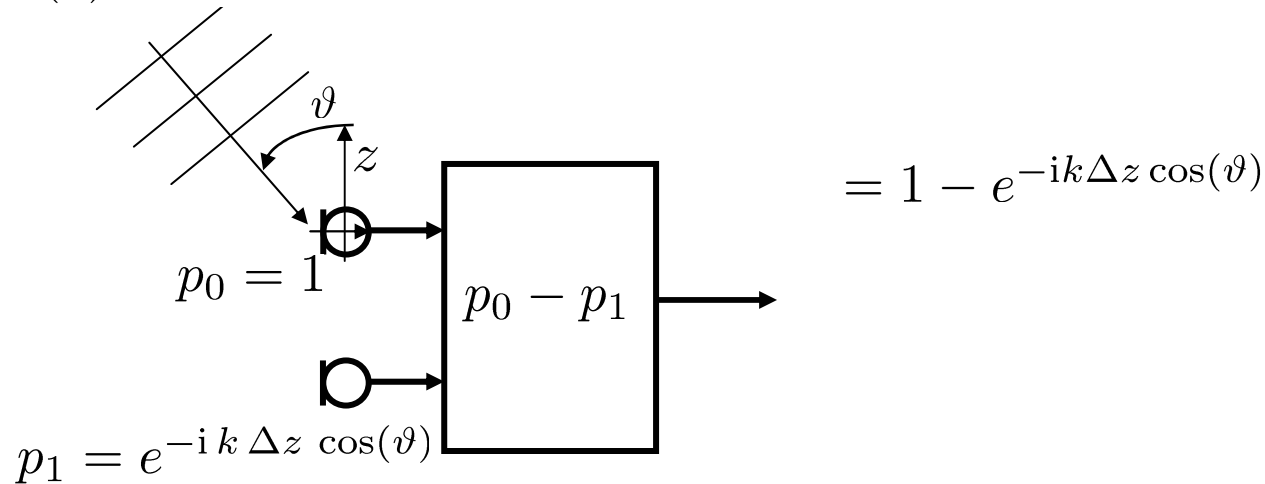
Soundfield Microphone

Zotter, Pomberger, 2010/2011

Der kleine Unterschied...

Differenz zweier Druckempfängersignale

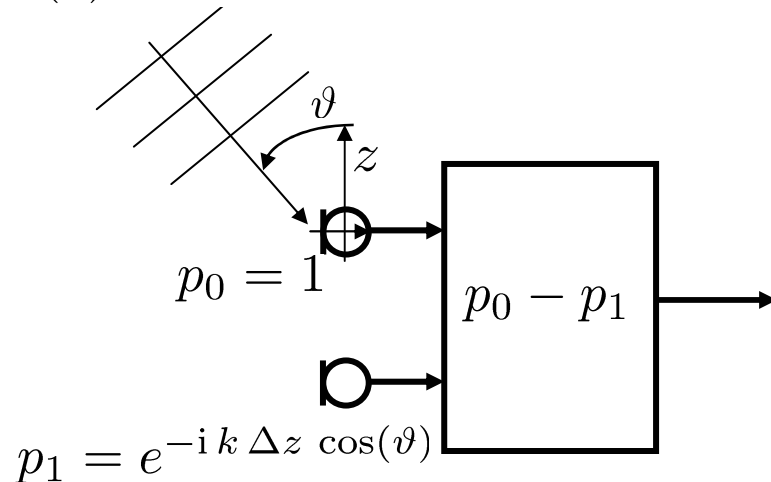
$$p(z) = e^{ikz \cos(\vartheta)} \quad \dots \text{ebene Welle}$$



Der kleine Unterschied...

Differenz zweier Druckempfängersignale

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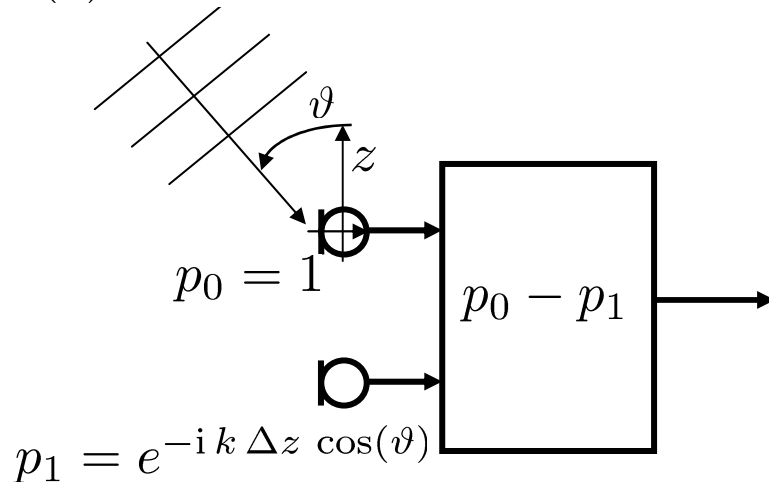


$$\begin{aligned} &= 1 - e^{-ik \Delta z \cos(\vartheta)} \\ &= \left[e^{ik \frac{\Delta z}{2} \cos(\vartheta)} - e^{-ik \frac{\Delta z}{2} \cos(\vartheta)} \right] e^{-ik \frac{\Delta z}{2} \cos(\vartheta)} \\ &= 2i \sin \left[k \frac{\Delta z}{2} \cos(\vartheta) \right] e^{-ik \frac{\Delta z}{2} \cos(\vartheta)} \\ &\approx ik \Delta z \cos(\vartheta) \end{aligned}$$

Der kleine Unterschied...

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Warum heißt das „Gradient“:

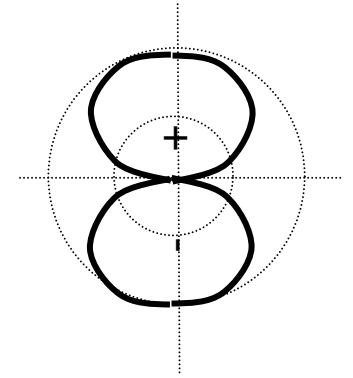
$$\frac{\partial}{\partial z} p|_0 = ik \cos(\vartheta)$$

$$\begin{aligned} &= 1 - e^{-ik \Delta z \cos(\vartheta)} \\ &= [e^{ik \frac{\Delta z}{2} \cos(\vartheta)} - e^{-ik \frac{\Delta z}{2} \cos(\vartheta)}] e^{-ik \frac{\Delta z}{2} \cos(\vartheta)} \\ &= 2i \sin[k \frac{\Delta z}{2} \cos(\vartheta)] e^{-ik \frac{\Delta z}{2} \cos(\vartheta)} \\ &\approx ik \Delta z \cos(\vartheta) \end{aligned}$$

Vereinfachungen:

$$\sin(ik \frac{\Delta z}{2} \cos(\vartheta)) \approx ik \frac{\Delta z}{2} \cos(\vartheta)$$

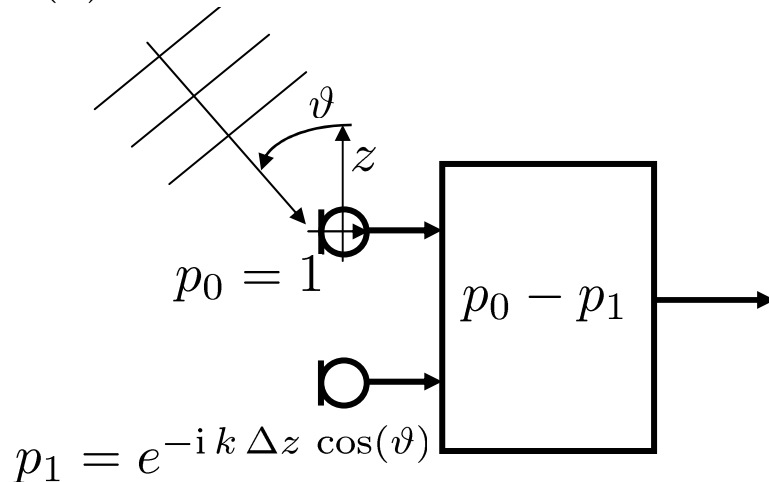
$$\cos(ik \frac{\Delta z}{2} \cos(\vartheta)) \approx 1$$



Der kleine Unterschied...

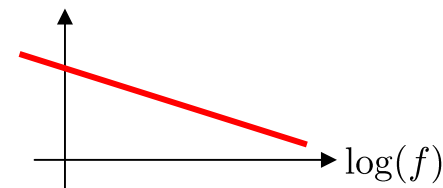
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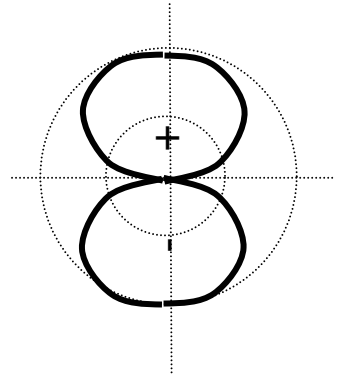
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Frequenzunabhängige Empfindlichkeit nach Filterung



Warum heißt das „Gradient“:

$$\frac{\partial}{\partial z} p|_0 = ik \cos(\vartheta)$$



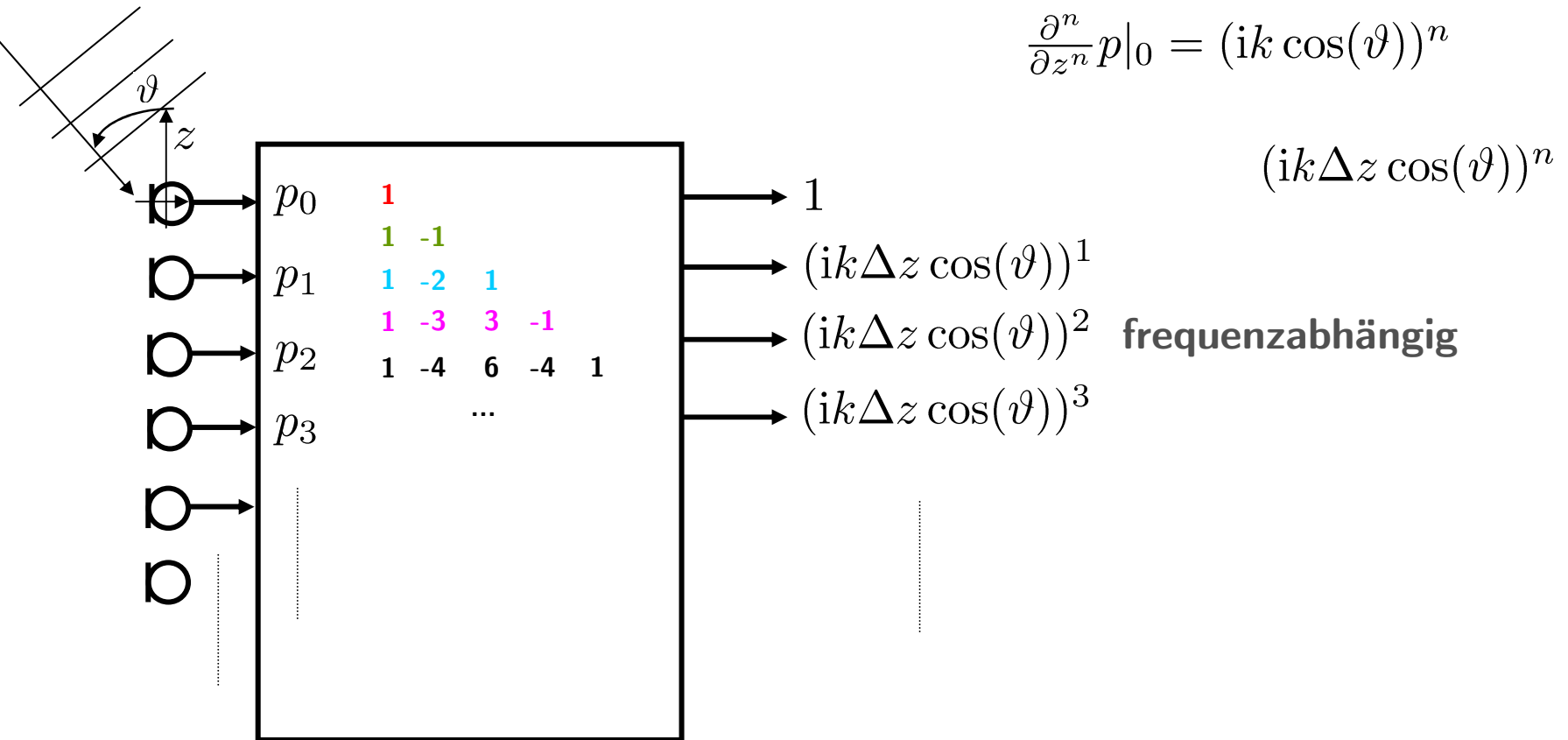
Elektronisch gebildete Gradienten aller Ordnungen

höhere Differenzen zwischen Druckempfängersignalen

$$p_l = e^{-i k l \Delta z \cos(\vartheta)} \quad \text{am } l\text{-ten Mikrofon}$$

Gradient n-ter Ordnung (Ziel)

$$\frac{\partial^n p}{\partial z^n} \Big|_0 = (i k \cos(\vartheta))^n$$



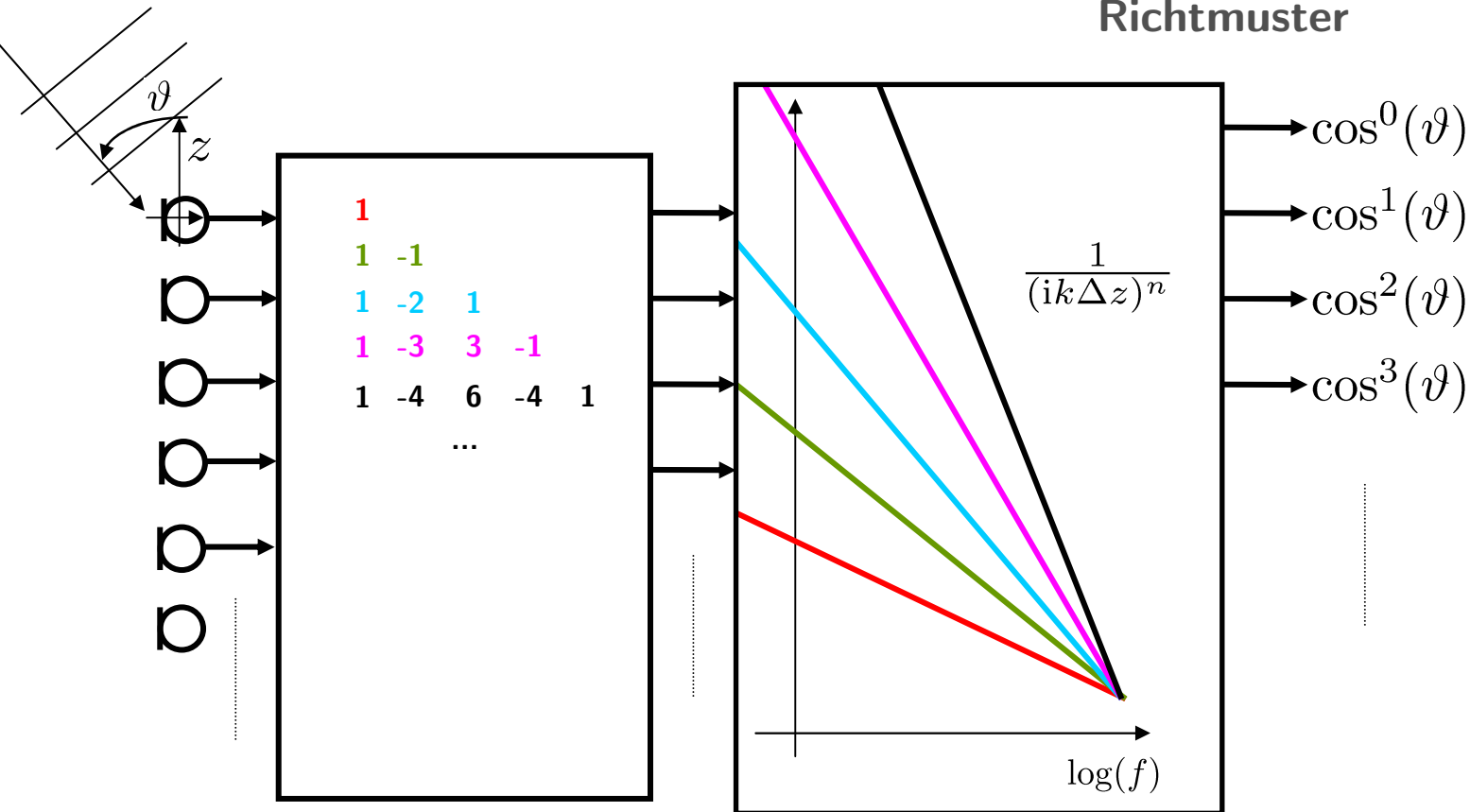
Olson 1946, JASA. Elko 2000-.

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frequenzunabhängige
Richtmuster



Olson 1946, JASA. Elko 2000-.

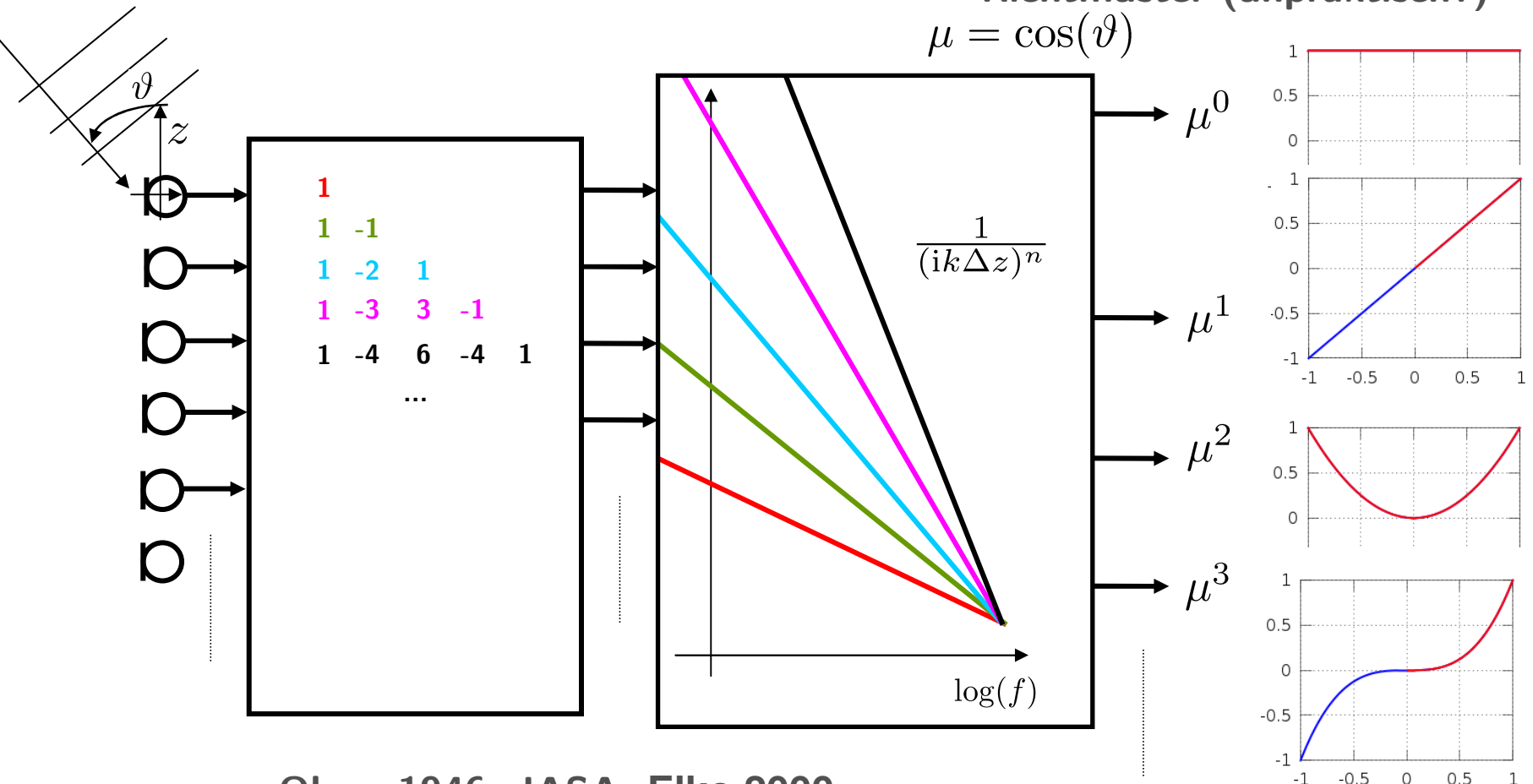
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frequenzunabhängige
Richtmuster (unpraktisch?)

$$\mu = \cos(\vartheta)$$



Olson 1946, JASA. Elko 2000-.

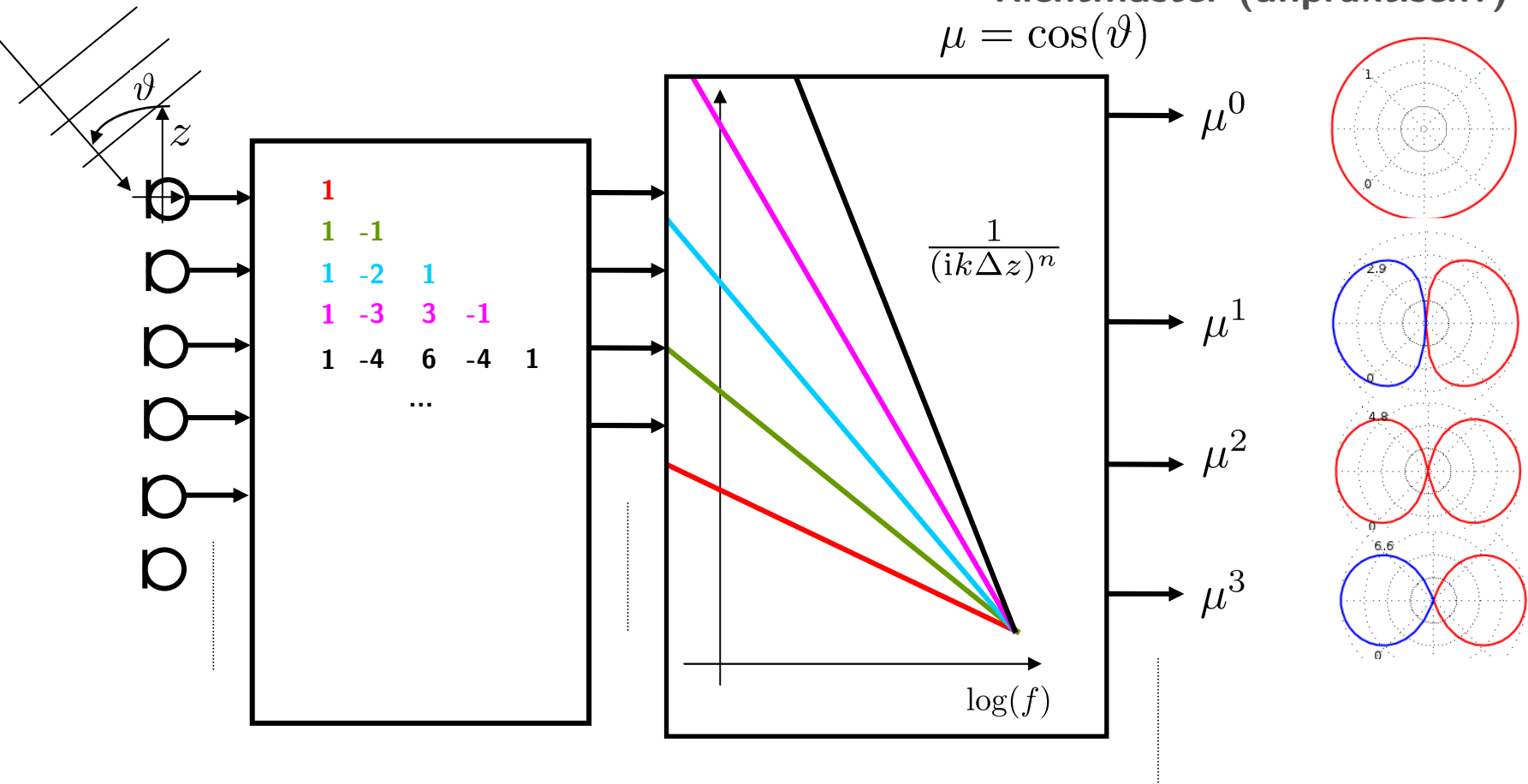
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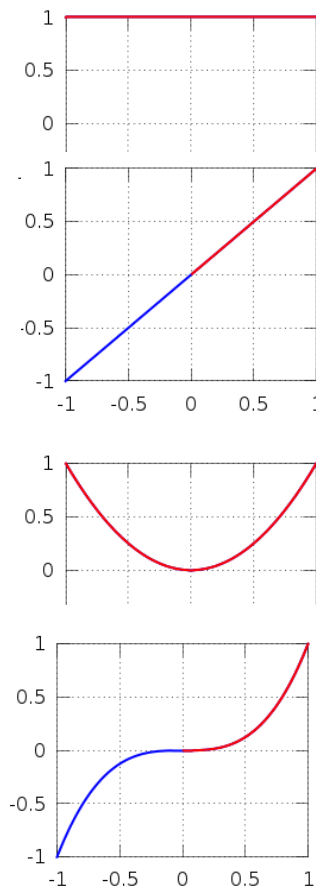
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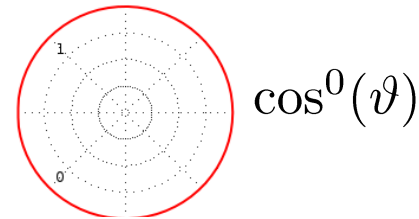
Olson 1946, JASA. Elko 2000-.

Polynome!

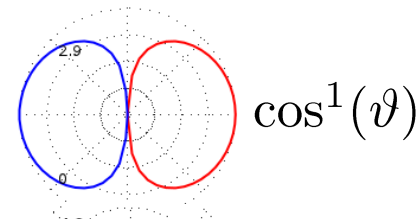
Wer kennt sich gut mit Polynomen und Potenzen aus?



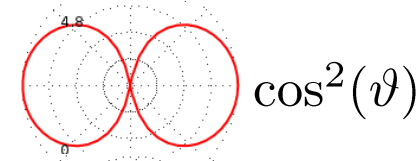
μ^0



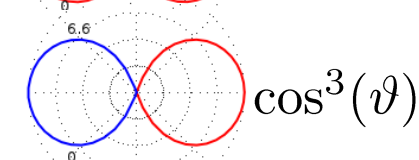
μ^1



μ^2



μ^3

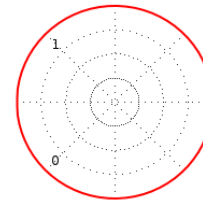


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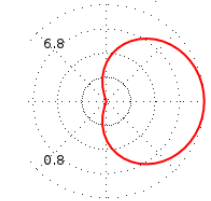
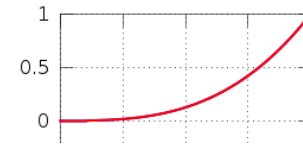
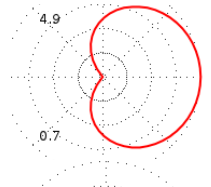
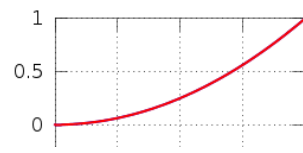
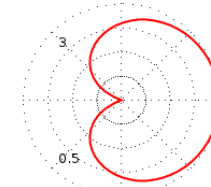
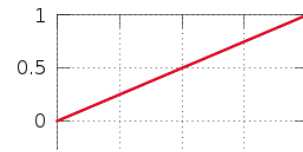
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$$(\mu + 1)^n / 2^n$$



$$(\cos \vartheta + 1)^n / 2^n$$

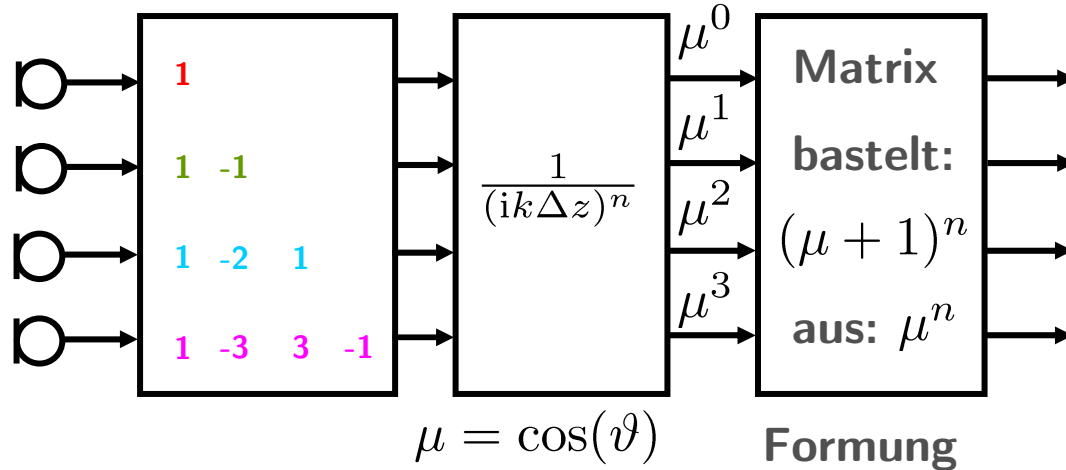


Jerome Daniel, 2001.

Eine Niere für jede Ordnung, in Ordnung?

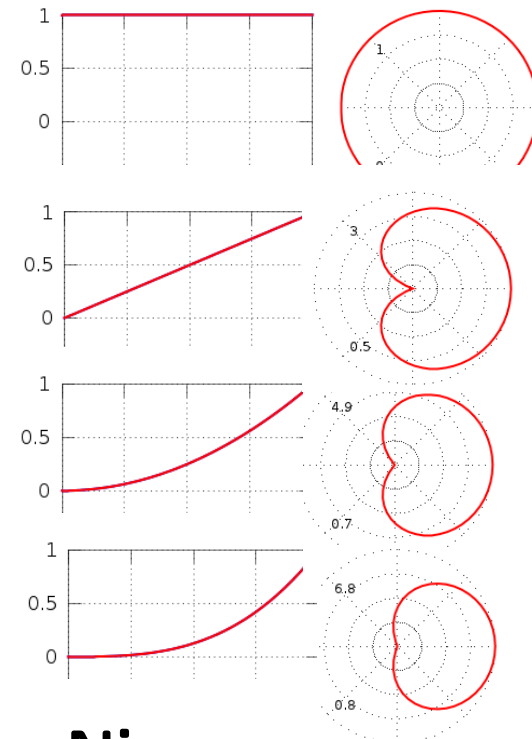
Nierenrichtwirkung:

maximal flache Nullstelle bei 180° ! (Butterworth)



$$(\mu + 1)^n / 2^n$$

$$(\cos \vartheta + 1)^n / 2^n$$



**Niere n-ter
Ordnung**

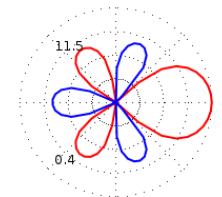
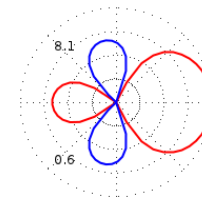
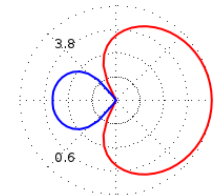
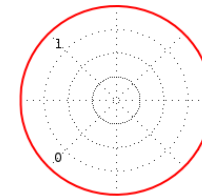
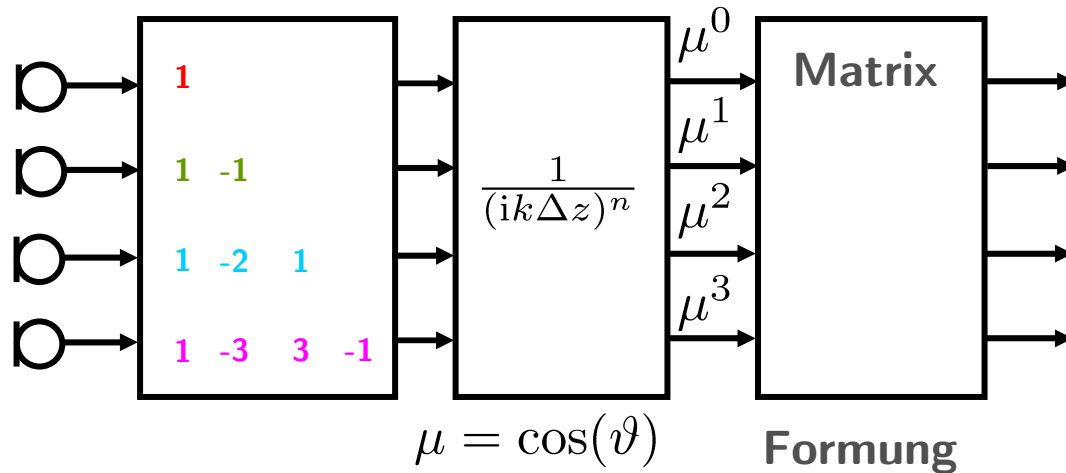
Jerome Daniel, 2001.

Die Nebenkeulen des verschobenen Tschebyscheffs

Richtwirkung:

konstante Nebenkeulenhöhe -10! (Tschebyscheff)

$$T_n[(\mu + 1)\alpha - 1] / T_n(2\alpha - 1)$$



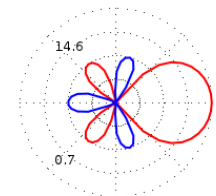
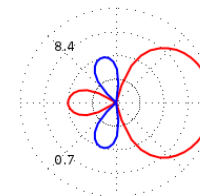
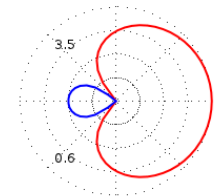
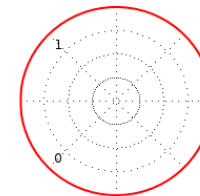
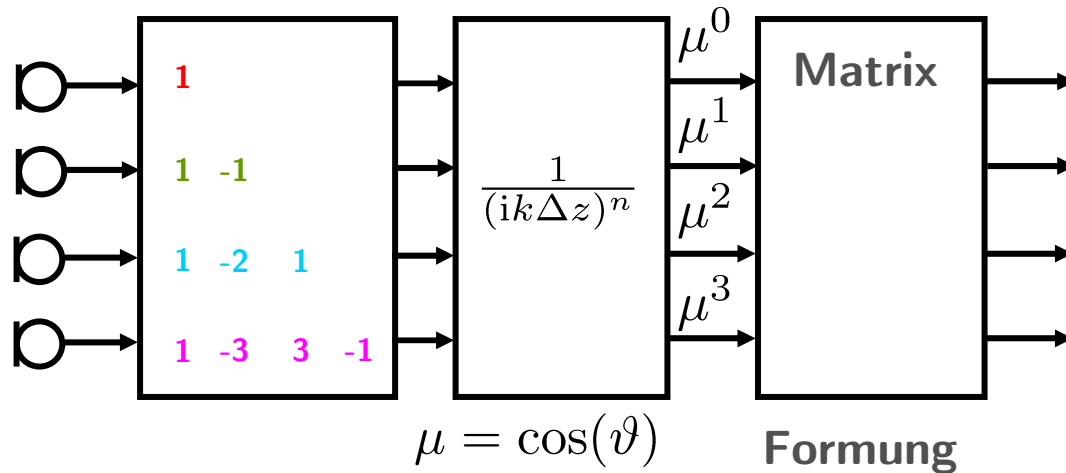
Elko, 2004; Koretz, Rafaely, 2009.

Die Nebenkeulen des verschobenen Tschebyscheffs

Richtwirkung:

konstante Nebenkeulenhöhe -15! (Tschebyscheff)

$$T_n[(\mu + 1)\alpha - 1] / T_n(2\alpha - 1)$$



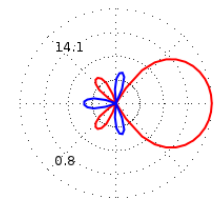
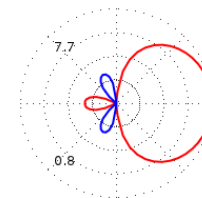
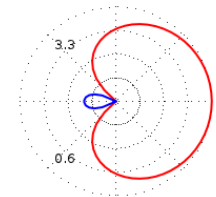
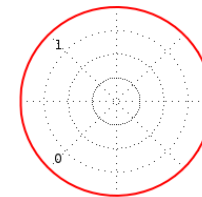
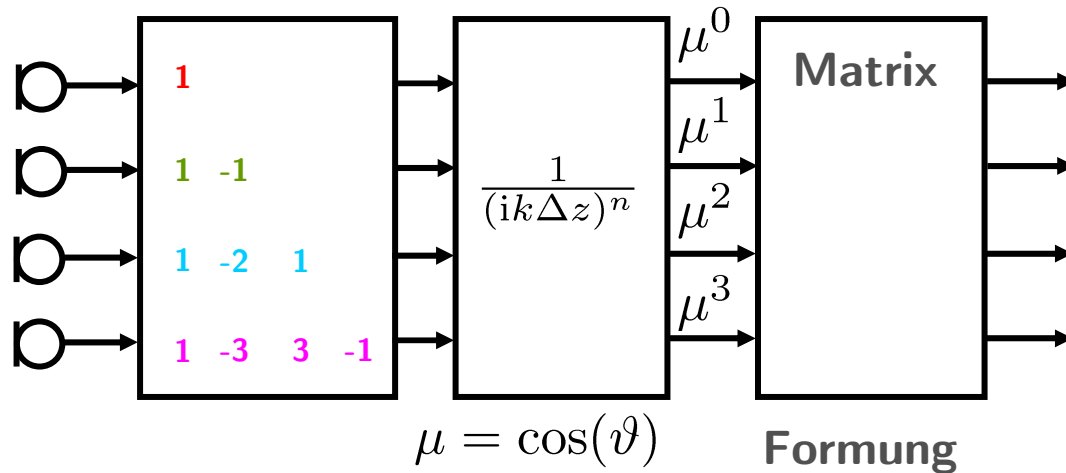
Elko, 2004; Koretz, Rafaely, 2009.

Die Nebenkeulen des verschobenen Tschebyscheffs

Richtwirkung:

konstante Nebenkeulenhöhe -20! (Tschebyscheff)

$$T_n[(\mu + 1)\alpha - 1] / T_n(2\alpha - 1)$$



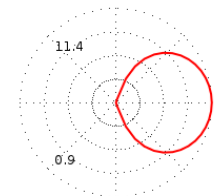
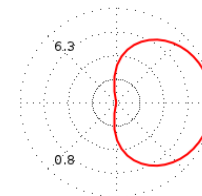
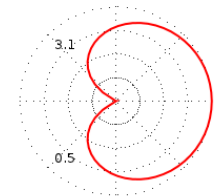
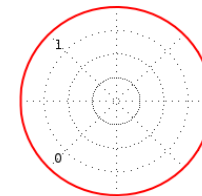
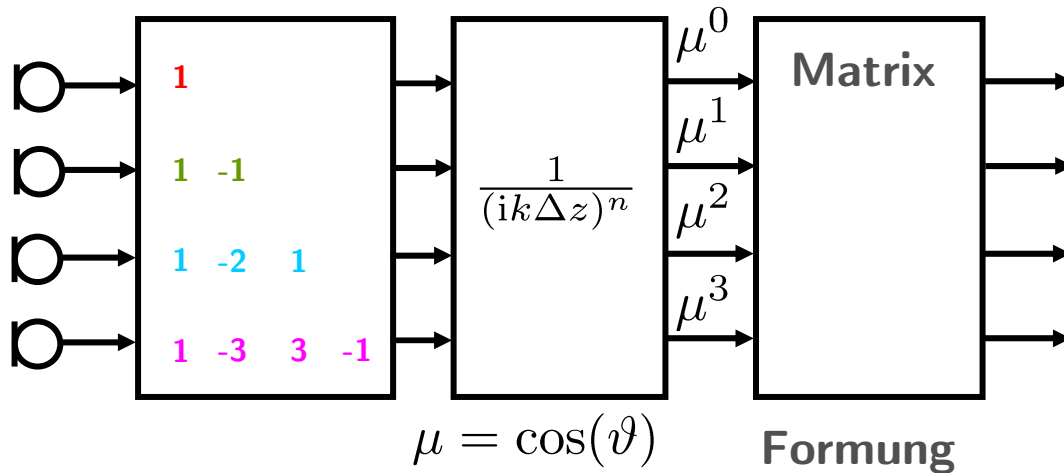
Elko, 2004; Koretz, Rafaely, 2009.

Die Nebenkeulen des verschobenen Tschebyscheffs

Richtwirkung:

konstante Nebenkeulenhöhe -30! (Tschebyscheff)

$$T_n[(\mu + 1)\alpha - 1] / T_n(2\alpha - 1)$$



Elko, 2004; Koretz, Rafaely, 2009.

Optimaler Richtfaktor

Definition:

Vergleich der quadrierten mittleren Richtungsempfindlichkeit mit jener eines richtungsunempfindlichen Mikrofons

$$\gamma = \frac{\int_{-1}^1 1^2 d\mu}{\int_{-1}^1 p(\mu)^2 d\mu}$$

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Maximaler Richtfaktor: $\int_{-1}^1 p(\mu)^2 d\mu \rightarrow \min.$
s.t. $p(\mu)|_1 = 1$

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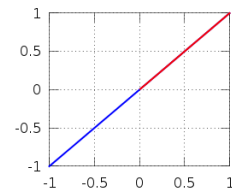
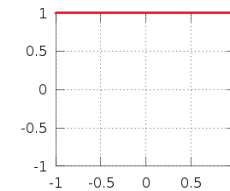
$$\sum_{n,n'} b_n b_{n'} \int_{-1}^1 \mu^n \mu^{n'} d\mu$$

Optimaler Richtfaktor

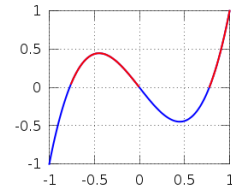
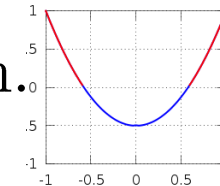
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Maximaler Richtfaktor: $\int_{-1}^1 p(\mu)^2 d\mu \rightarrow \min.$
 $p(\mu) = \sum_n a_n P_n(\mu)$ s.t. $p(\mu)|_1 = 1$



Legendre-Polynome $\int_{-1}^1 P_n(\mu) P_{n'}(\mu) d\mu = \frac{2 \delta_{n,n'}}{2n+1}$ **orthogonal**

$$P_0 = 1$$

$$P_1 = \mu$$

$$P_2 = \frac{3}{2}\mu^2 - \frac{1}{2}$$

$$P_3 = \frac{15}{8}\mu^3 - \frac{9}{4}\mu$$

...

$$P_{n+1} = \frac{2n+1}{n+1}\mu P_n - \frac{n}{n+1}P_{n-1}$$

Optimaler Richtfaktor

Definition:

Vergleich der quadrierten mittleren Richtungsempfindlichkeit mit jener eines richtungsunempfindlichen Mikrofons

$$\gamma = \frac{\int_{-1}^1 1^2 d\mu}{\int_{-1}^1 p(\mu)^2 d\mu}$$

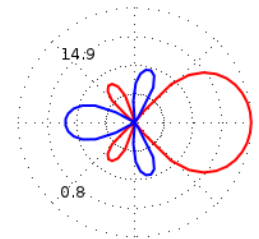
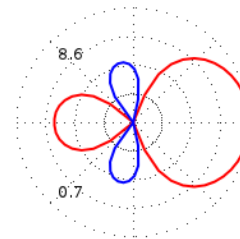
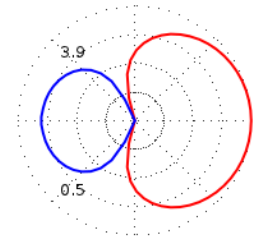
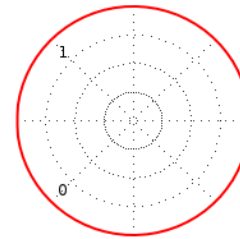
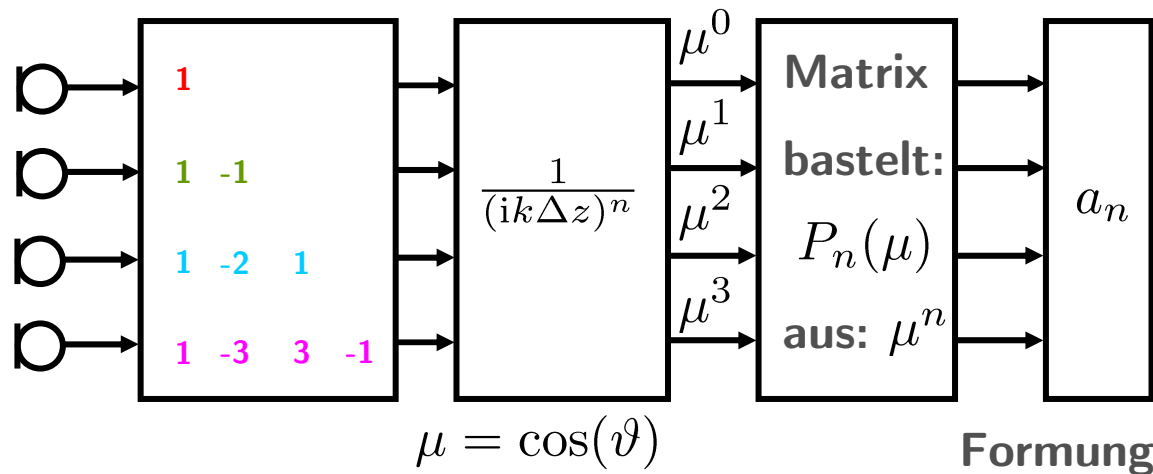
Maximaler Richtfaktor: $\sum_n \frac{a_n^2}{2n+1} \rightarrow \min.$
 $p(\mu) = \sum_n a_n P_n(\mu)$ s.t. $\sum_n a_n = 1$

$$\implies a_n = \frac{2n+1}{(N+1)^2}$$

Gib Diffusschall keine Chance!

Hyperniere:

Richtwirkung mit maximalem Richtfaktor



Formung

Hyperniere n-ter Ordnung

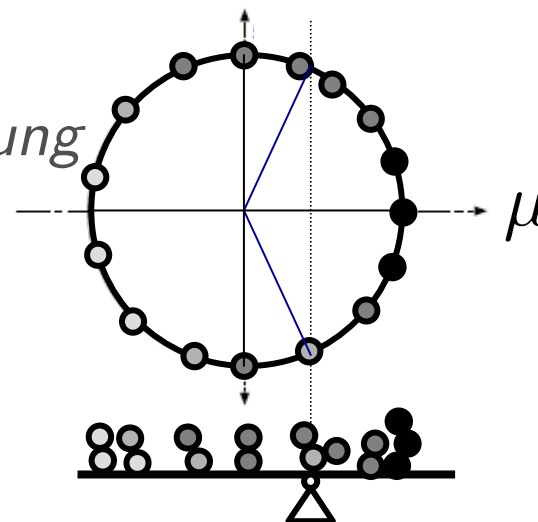
störend: Bündelung nach hinten

Balancieren zwischen vorne und hinten

Definition:

*Schwerpunkt der quadrierten
Richtungsempfindlichkeit auf der Zielrichtung*

$$\sigma = \frac{\int_{-1}^1 \mu p(\mu)^2 d\mu}{\int_{-1}^1 p(\mu)^2 d\mu}$$



maximaler Schwerpunkt:

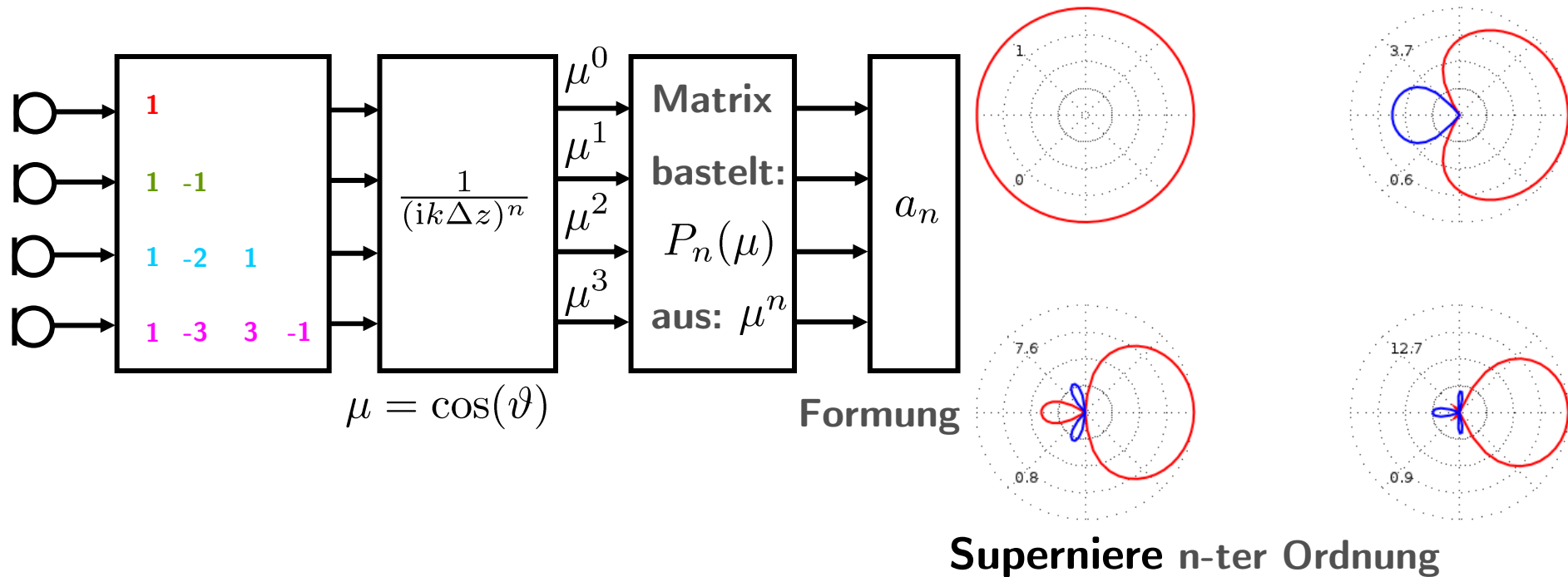
$$\frac{\sum_n n a_n a_{n-1}}{\sum_n a_n^2} \rightarrow \max.$$

$$\implies a_n = P_n \left[\cos\left(\frac{137.9^\circ}{N+1.52}\right) \right]$$

Jerome Daniel, 2001 (rE-Vektor); + eigene Approximation

Supernieren sind super in jeder Ordnung!

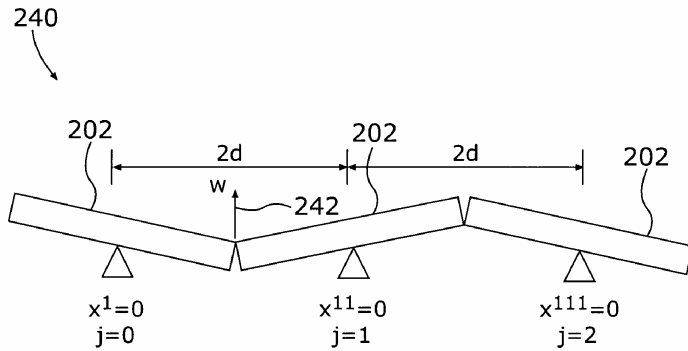
Superniere:
maximaler Vorne-Hinten-Schwerpunkt



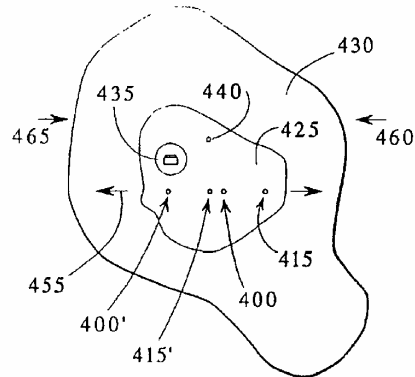
Jerome Daniel, 2001.

Akustische Gradientenempfänger höherer Ordnung

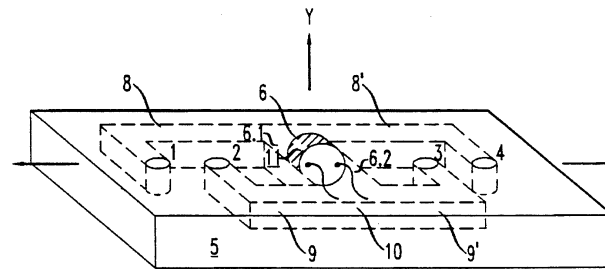
Patente



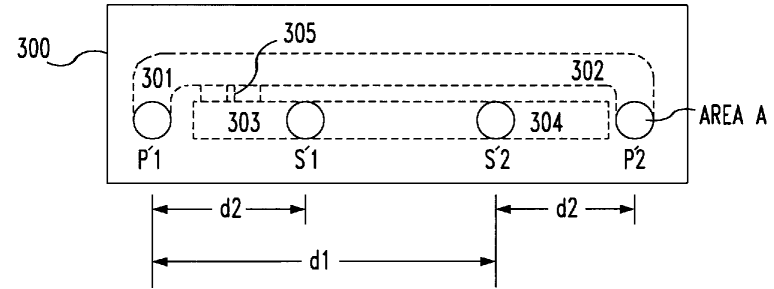
Miles 2005



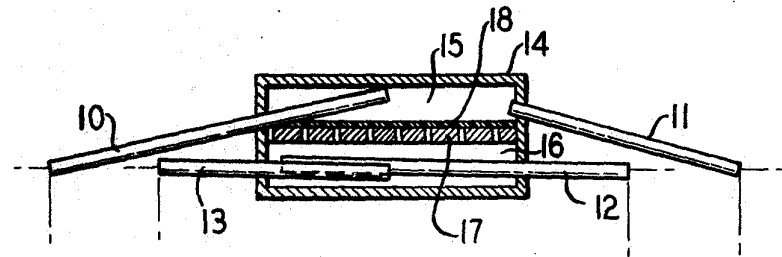
Killion 2006



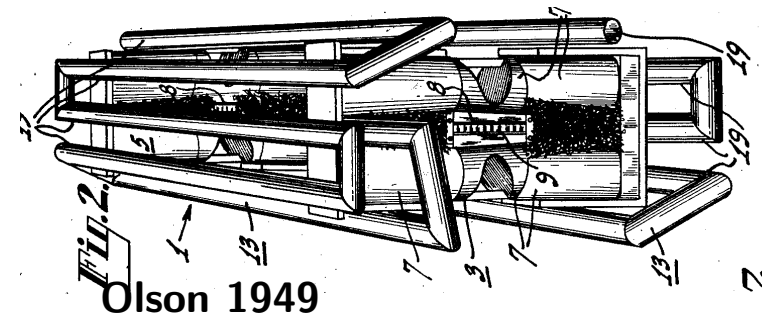
Bartlett/Zuniga 1996



Allen 2005



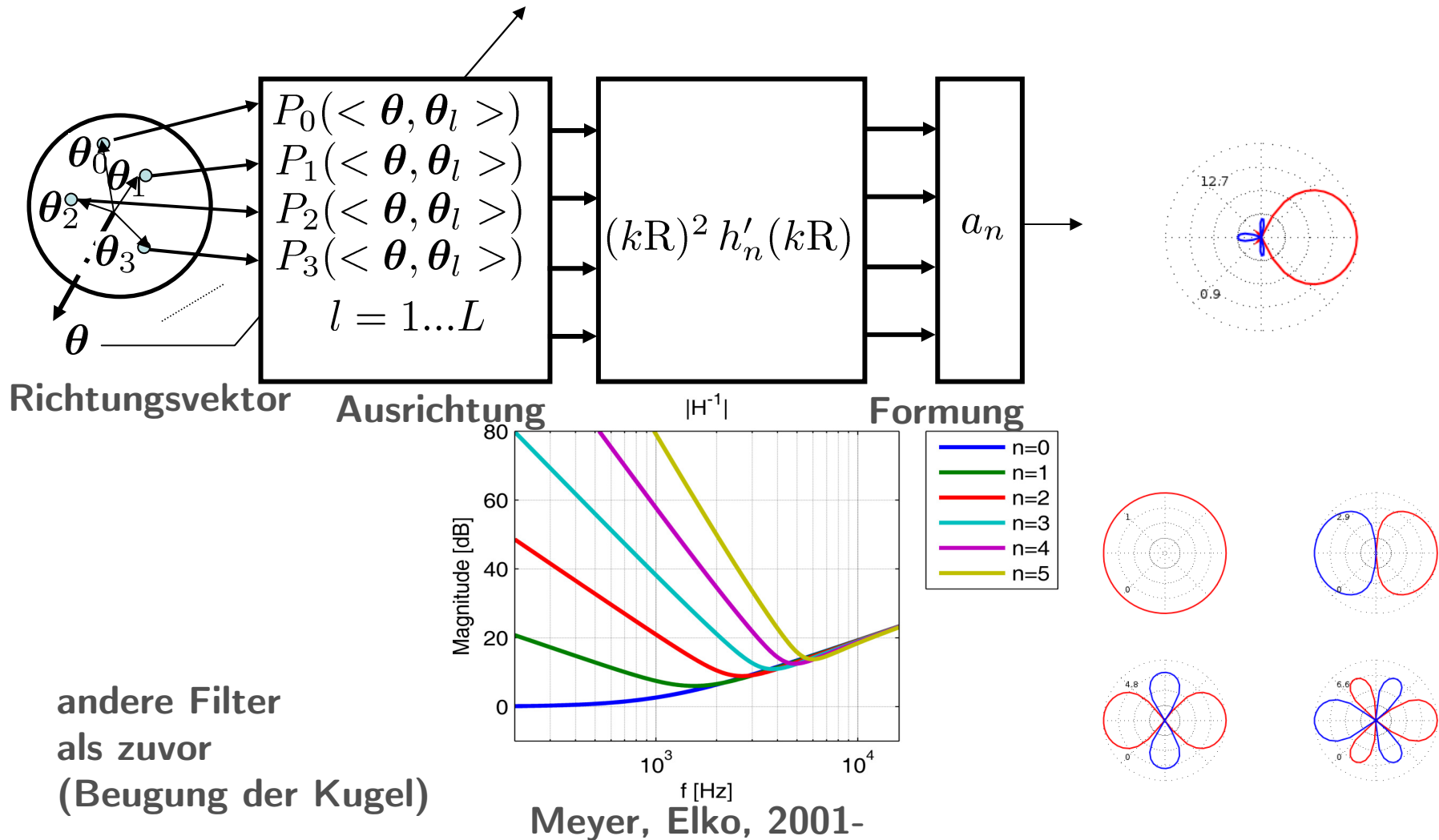
Sessler/West 1973



Olson 1949

Freies Ausrichten der Richtungsempfindlichkeit?

günstig verteilte Mikrofone auf schallharter Kugel

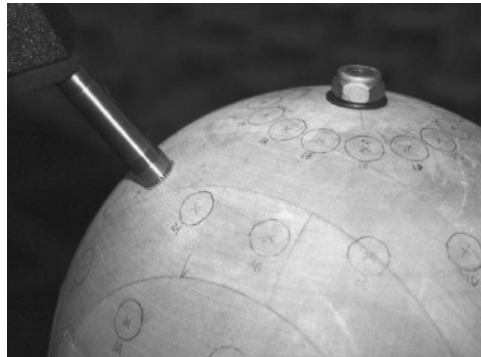


Higher order microphone arrays

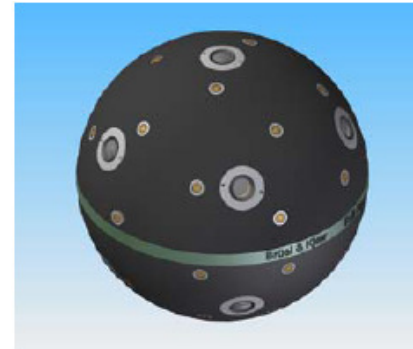
Meyer, Elko
2000-



Rafaely&Park,
2004-



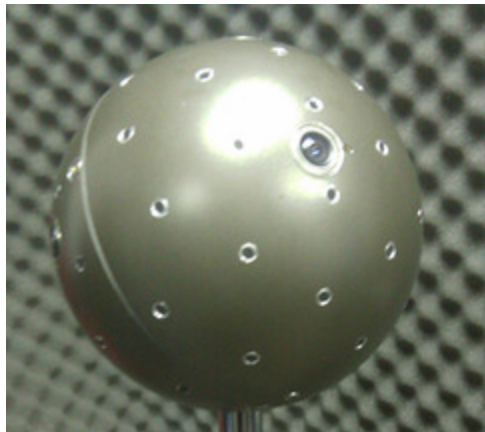
Petersen, Hald,
Moller-Juhl,
B&K, 2003-



Takashima, Nakagawa,
Williams, 01db, 2008



Li, Duraiswami, O'Donovan, Grassi,
visisonics, 2004-

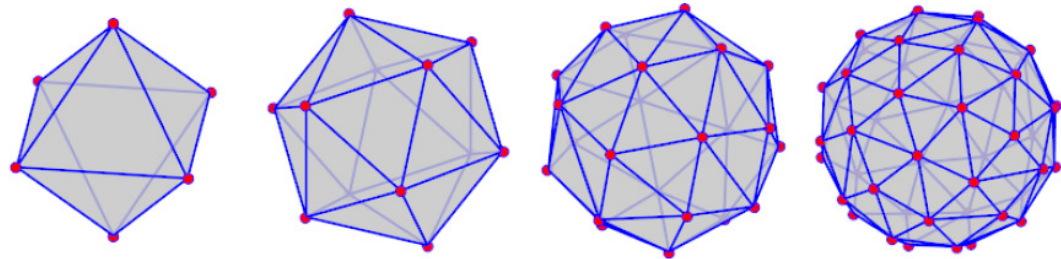


Jin, v.Schaik,
2006-



Günstige Abtastung der Kugel

- Spherical t-designs



Literatur über t-designs: Hardin+Sloan „The Improved Snub-Cube ...“

$$\int_{\mathbb{S}^2} P_t(\langle \boldsymbol{\theta}, \boldsymbol{\theta}_s \rangle) d\boldsymbol{\theta}_s = \frac{4\pi}{L} \sum_{l=1}^L P_t(\langle \boldsymbol{\theta}, \boldsymbol{\theta}_l \rangle)$$

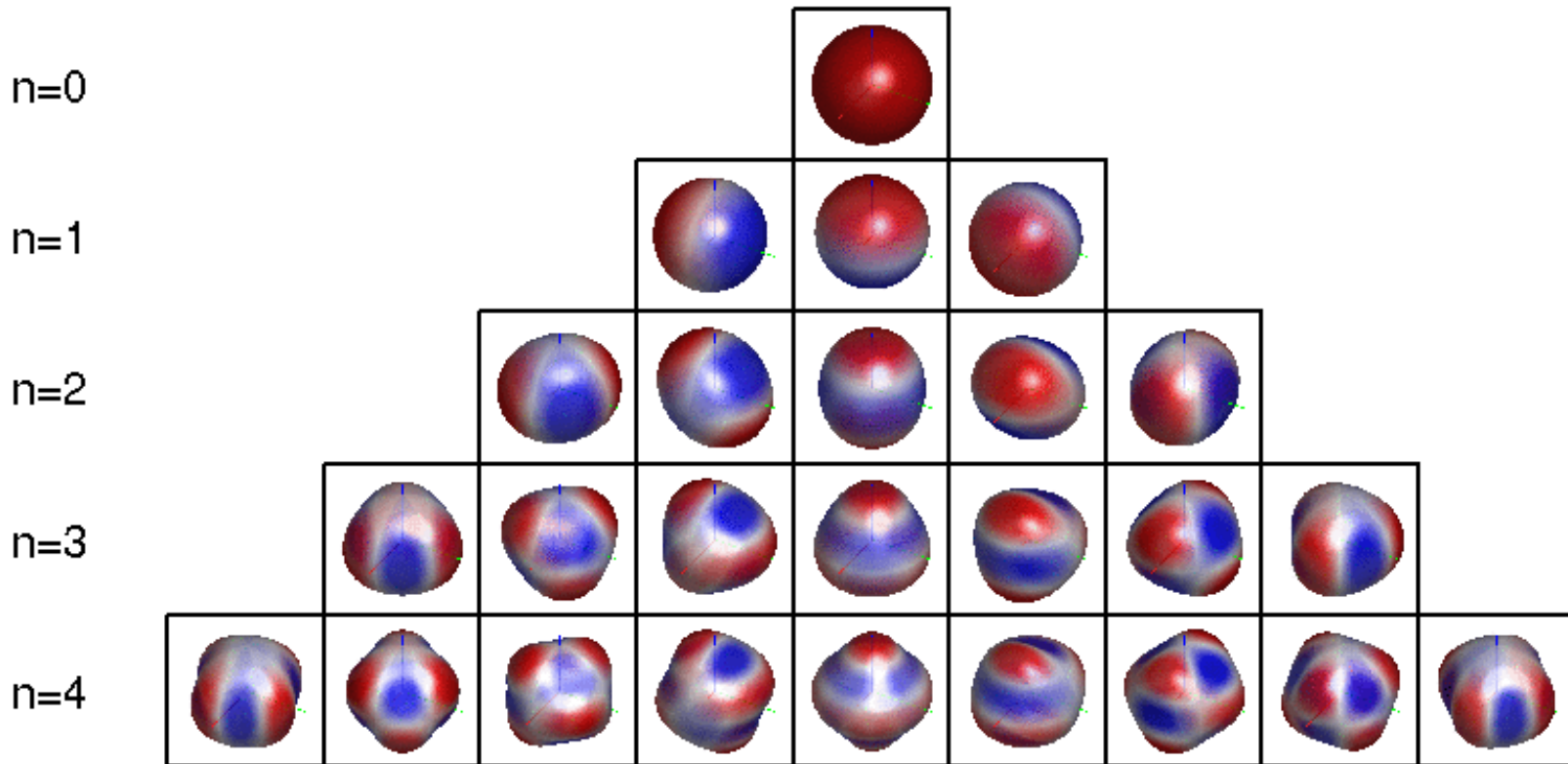
Wenn alle $\boldsymbol{\theta}_l$ gemeinsam ein t-design bilden:
kann Integral über alle Richtungen durch Summe
über $\boldsymbol{\theta}_l$ ersetzt werden.

Flexibler als rotationssymmetrisch:

Additionstheorem für Kugelflächenfunktionen

$$P_n(\langle \boldsymbol{\theta}, \boldsymbol{\theta}_s \rangle) = 2\pi \sum_{m=-n}^n Y_n^m(\boldsymbol{\theta}) Y_n^m(\boldsymbol{\theta}_s)$$

m=-4 m=-3 m=-2 m=-1 m=0 m=1 m=2 m=3 m=4

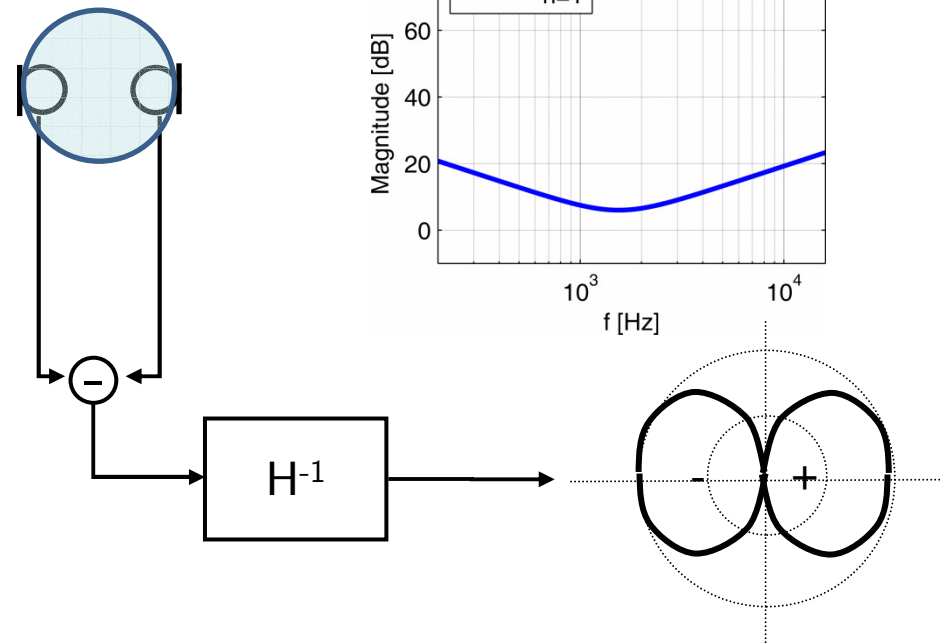
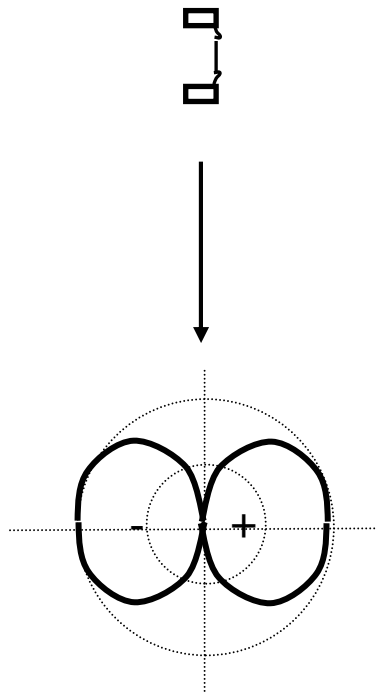


Allgemein

Pressure Gradient - 1.Order

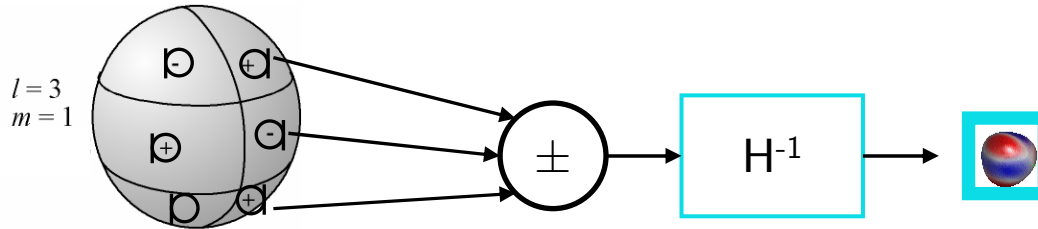
acustical

electronical

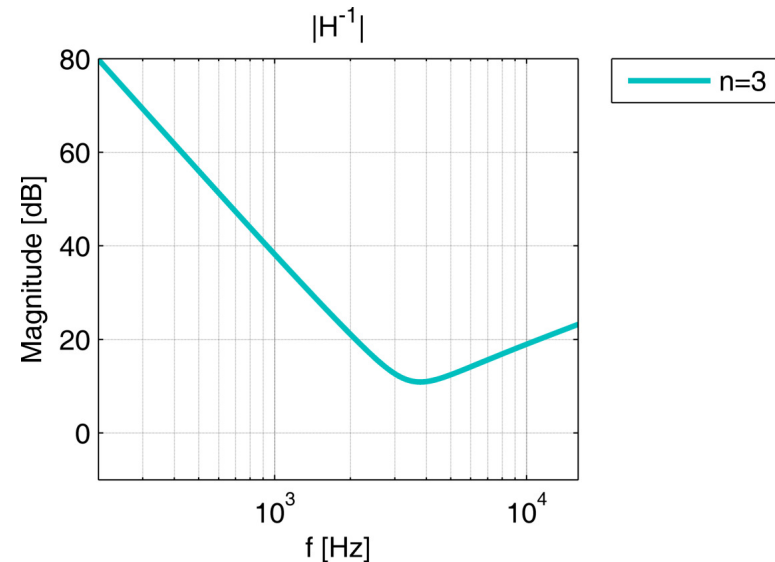
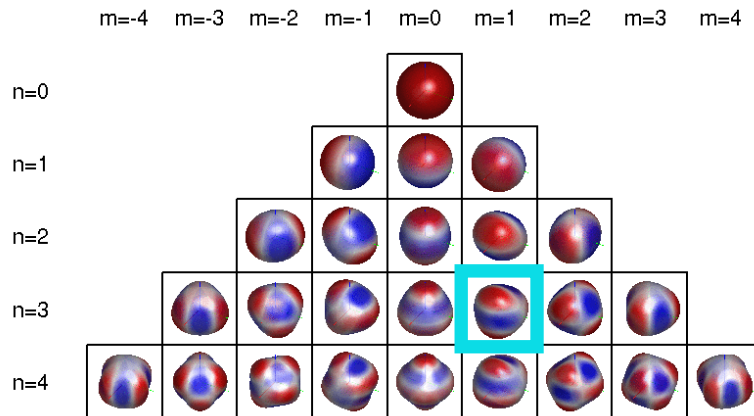


Vgl. Meyer, Elko, 2001- Vortrag Pomberger/Zotter, 2010

Allgemein

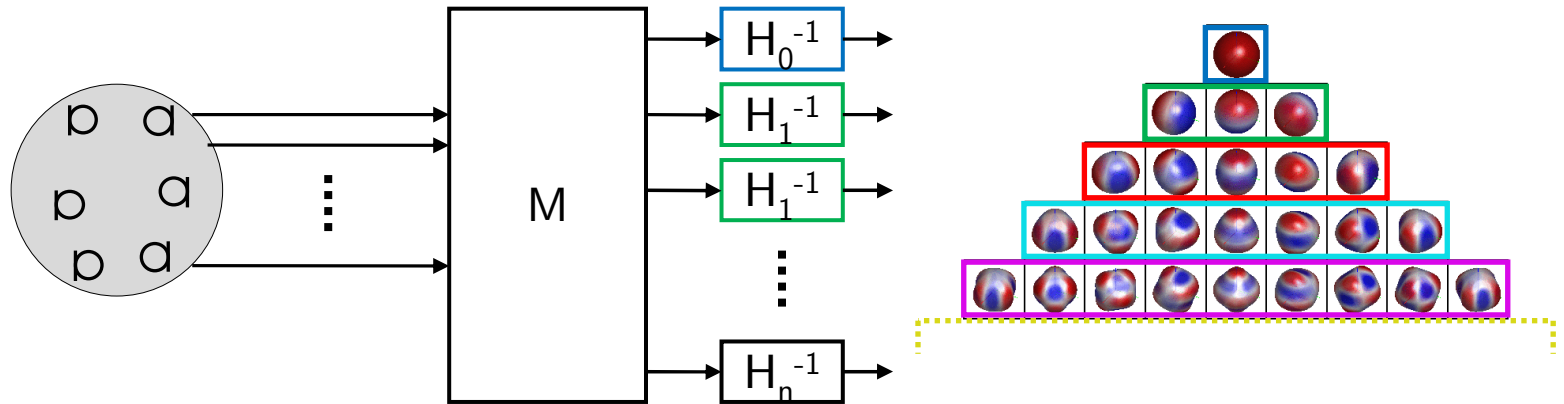


(vereinfacht)

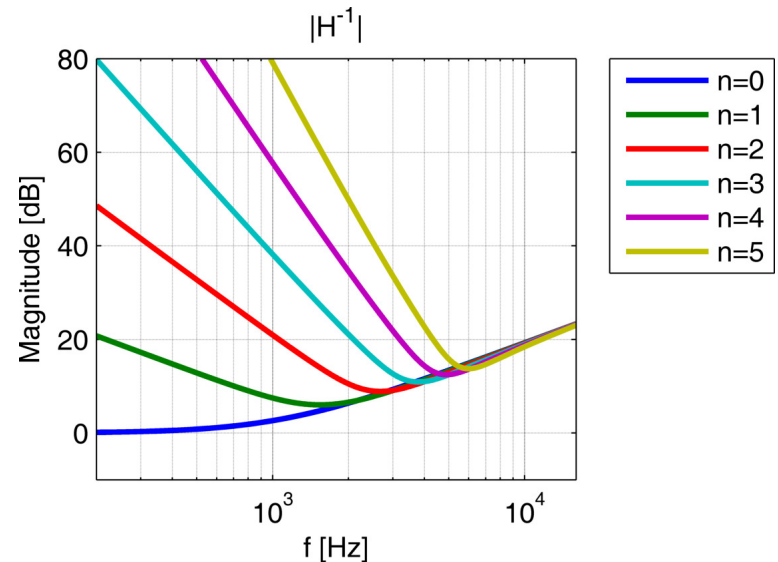


Vgl. Meyer, Elko, 2001- Vortrag Pomberger/Zotter, 2010

Allgemein



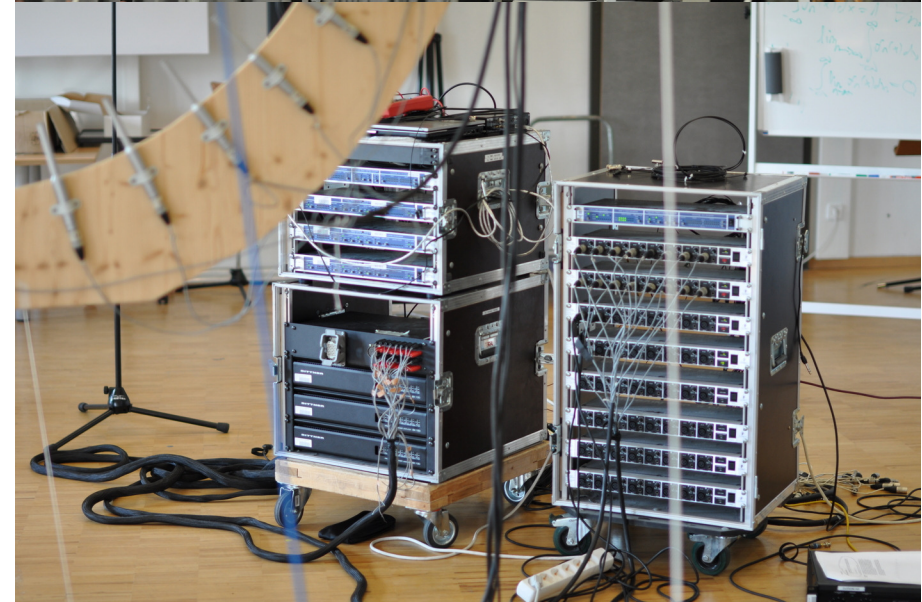
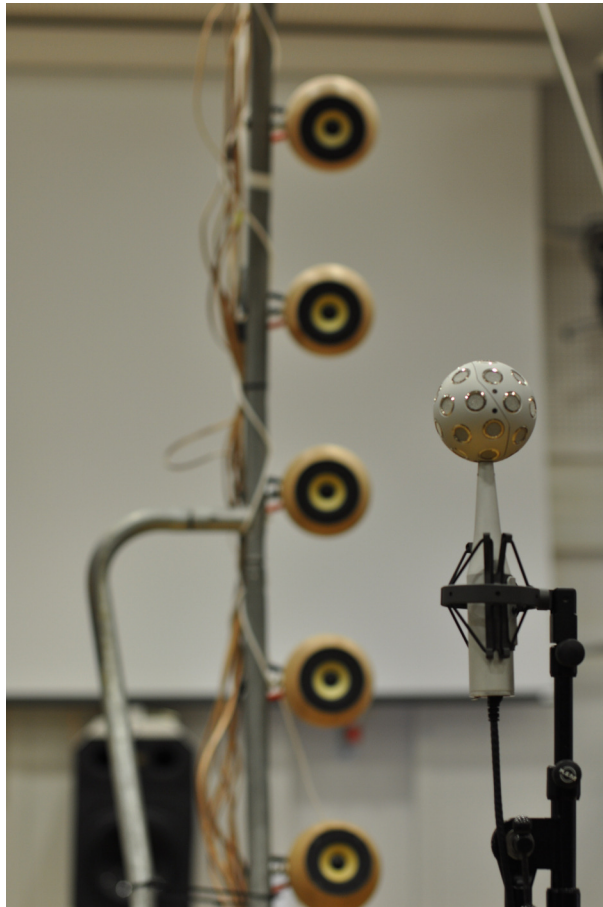
Holographic filters:



Vgl. Meyer, Elko, 2001- Vortrag Pomberger/Zotter, 2010

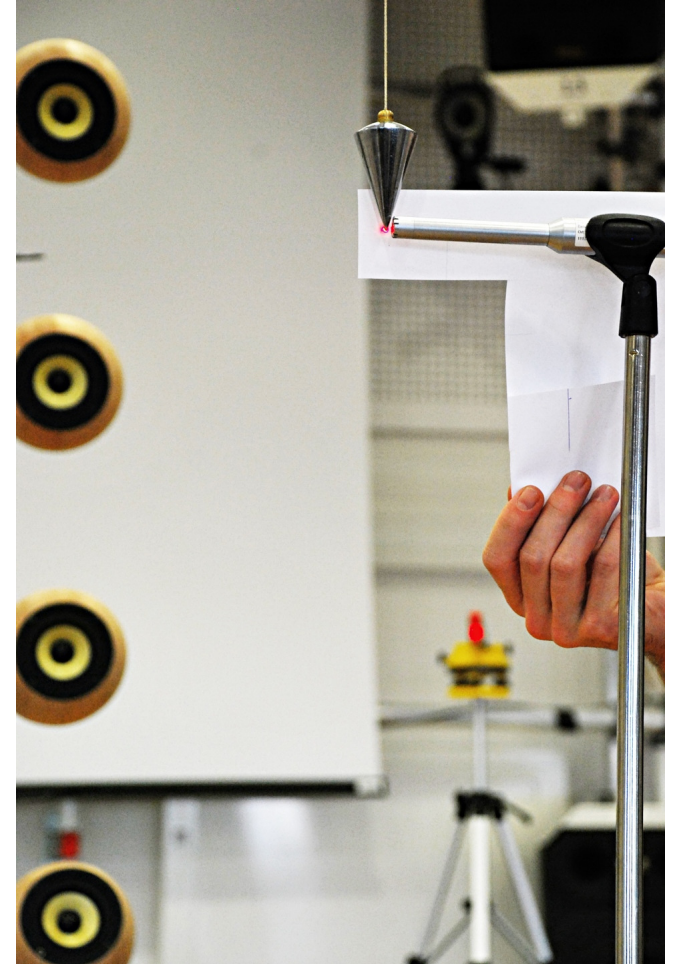
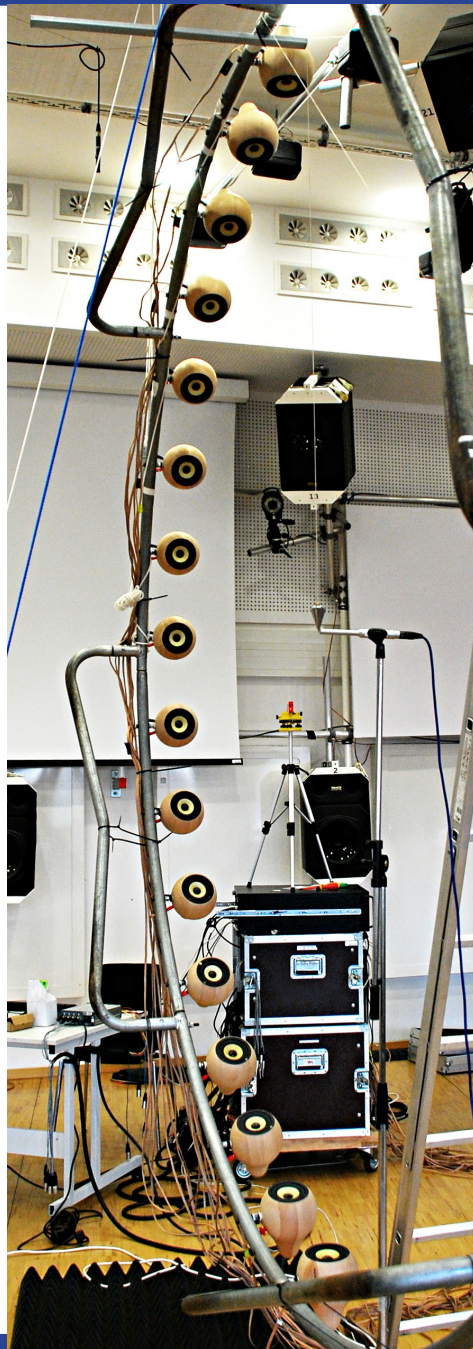
In der Praxis ist die Theorie eine andere...

Messung



Messung

Zentrierung



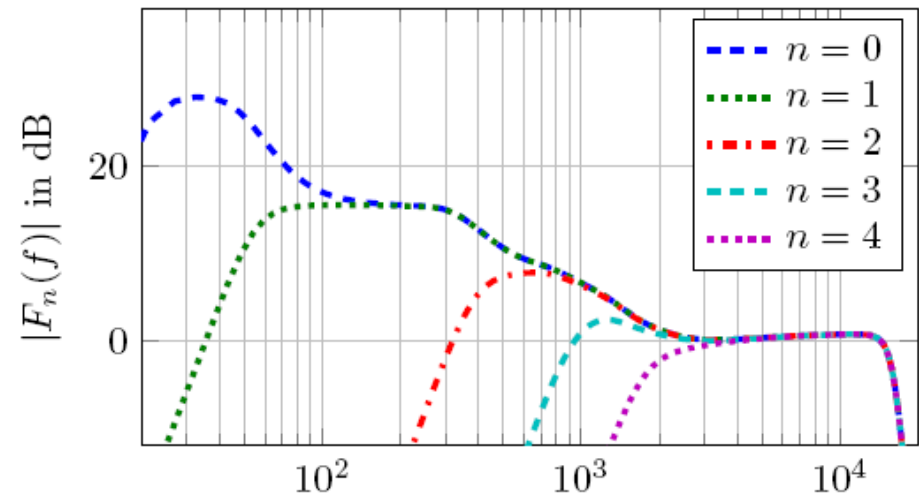
In der Praxis ist die Theorie eine andere...

Differenzen hoher Ordnung bei tiefen Frequenzen

$$1-1 = 0$$

$$1.01-1 = 0.01$$

-Verstärkung um 40dB?



(ICSA 2011, Baumgartner et al)

Kugelförmige Arrays



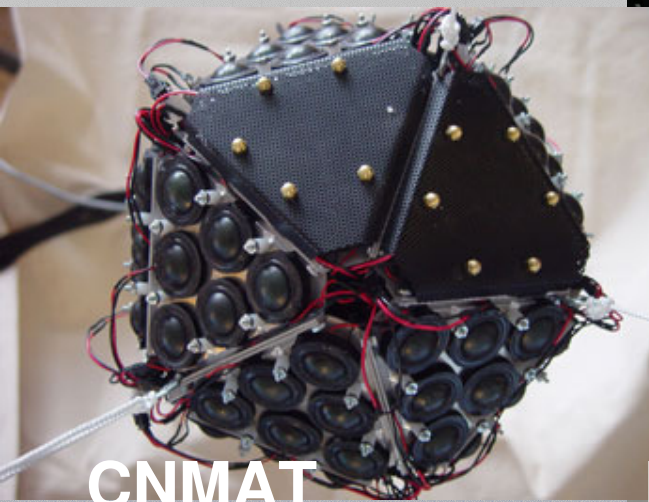
IRCAM



ITA/RWTH



IEM/KUG



CNMAT



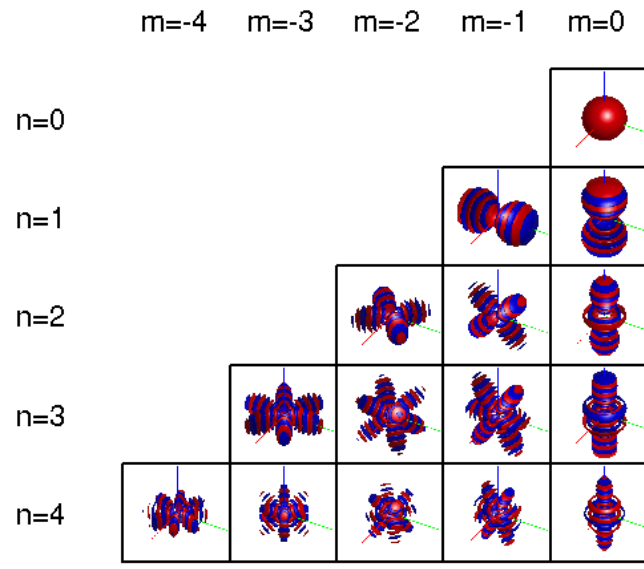
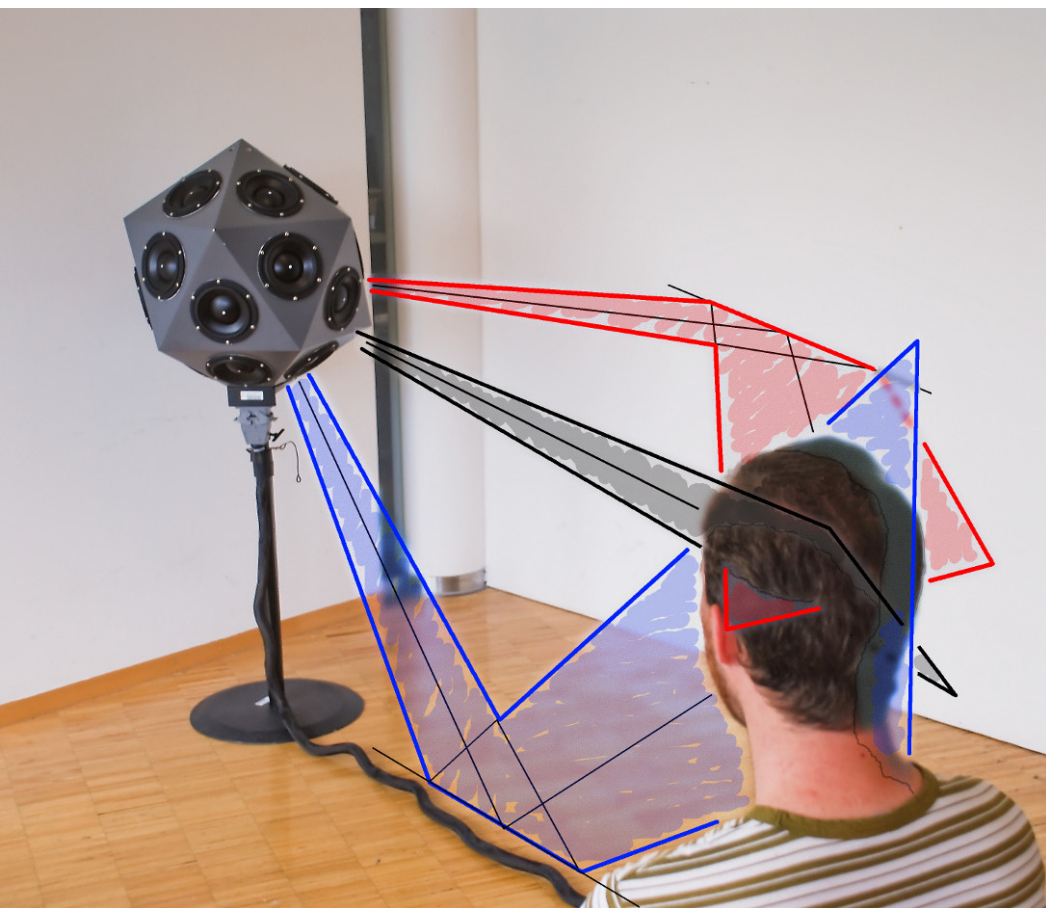
Princeton / PLork



OUTLINE

Abstrahlungsanalyse & -synthese (4/5)

Abstrahlungswiedergabe mit Kugellautsprecheranordnungen

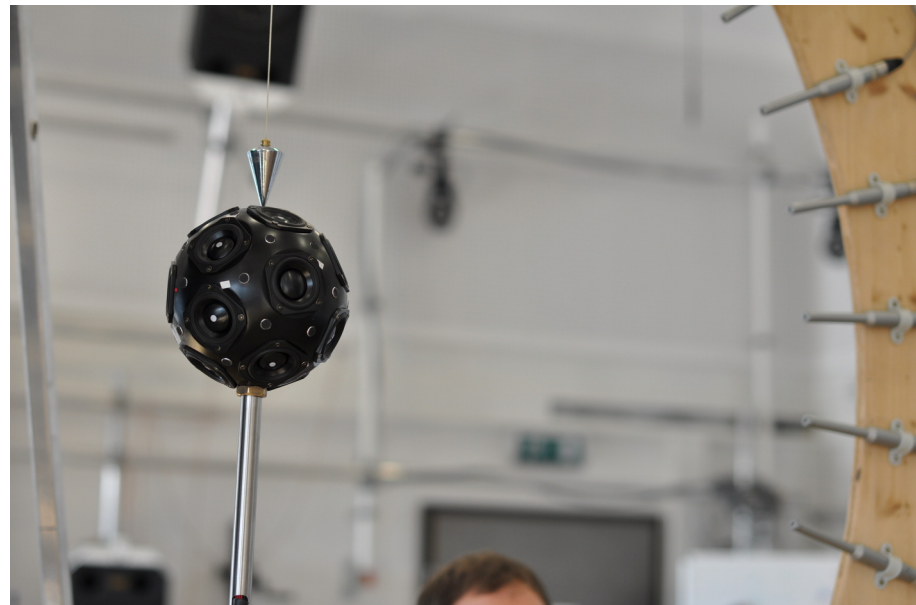


Zotter, 2010



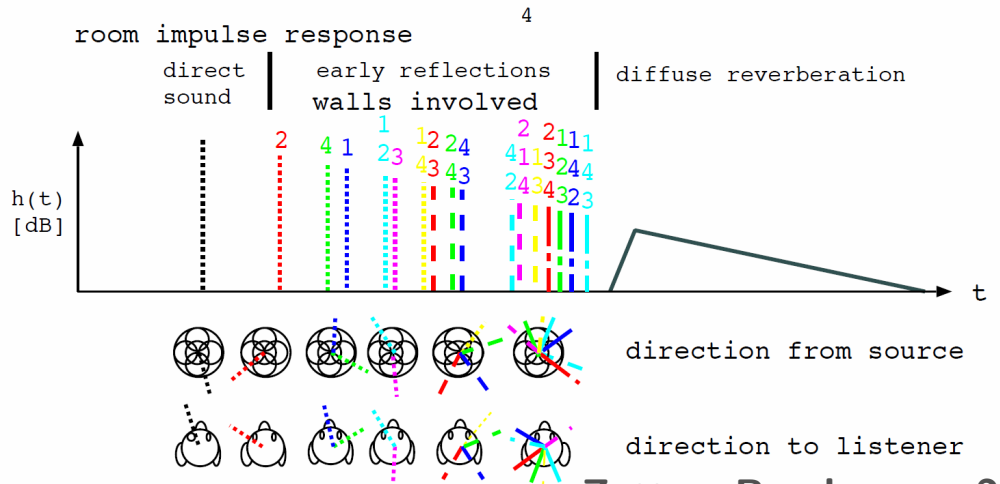
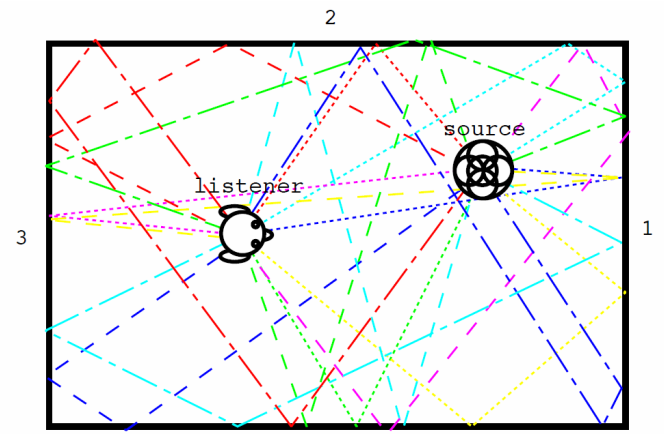
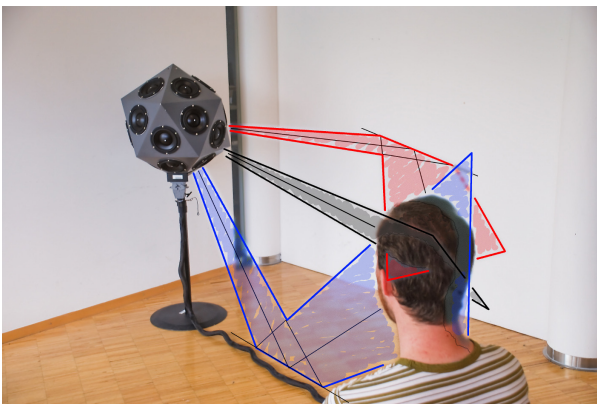
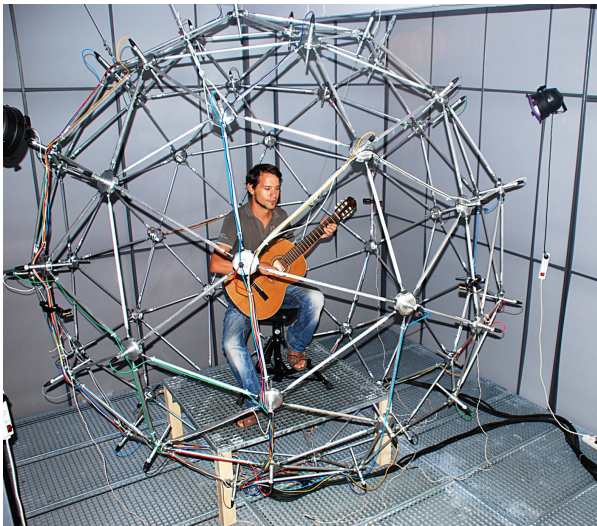
Messung Kugellautsprecher

(MATLAB GUI)



Abstrahlungsanalyse & -synthese (5/5)

Kombination aus Kugelmikrofon- und Kugellautsprecheranordnung im Raum



Zotter, Pomberger, 2009

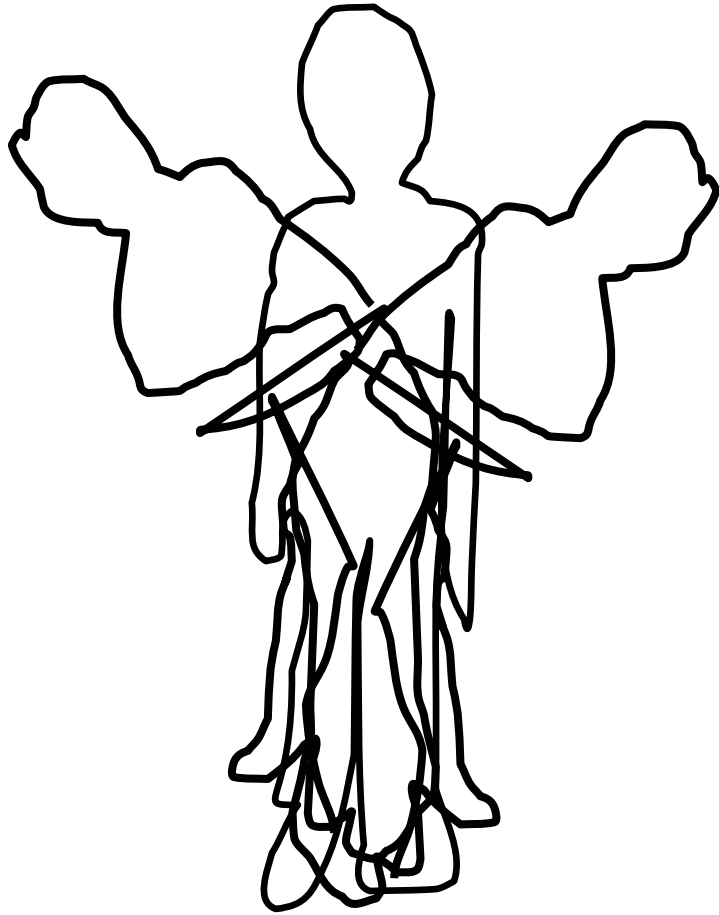


Offene Fragen

Wie ist es möglich,

- ausreichend kleine Anordnungen zu bauen, um hohe Frequenzen fehlerfrei abzubilden,
 - gleichzeitig ausreichendes SNR bei tiefen Frequenzen (Filterverstärkungen!) zu erhalten,
 - Gleichzeitig eine möglichst konstante Richtwirkung zu bekommen?
- Kurz gesagt: Wie bekommen wir möglichst große **Bandbreite**?

Dennoch: Genauigkeitsvorteil gegenüber rein akustischer Lösung.



Vielen Dank!