Spatial transformations for the enhancement of Ambisonic recordings

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Abstract

Spatial audio productions for circular or spherical surround playback facilities often use Ambisonics because of the smoothness it offers for panning and because of its classical and new main microphone array technology. In contrast to channel-based standards in surround sound, Ambisonics ideally offers flexibility regarding the loudspeaker setup around the listening area. Well-designed decoders should yield a spatial perspective that is largely independent of this setup. With the increasing availability of microphones capable of recording higher order Ambisonics, various transformations in the Ambisonic domain are desirable and necessary for enhancements during post production and playback. Alteration of source positions or their loudness levels can easily be done by transformations applied in the angular domain, of which a naive but impractical realisation would be to set up the playback loudspeakers differently than those specified in the decoder. Fortunately, corresponding transformations can always be performed as matrix operation in the Ambisonic signal domain, which, however, is currently not described well. In particular, such alterations are only well-described for first-order Ambisonics, while we lack systematic descriptions for the higher Ambisonic orders. This work presents ready-to-use implementations for the warping of the recording perspective and directional loudness modification of higher-order Ambisonics. What is more, Ambisonic mastering has only been done by ear in the past, wherefore this paper introduces a metering tool for monitoring the directional loudness levels of Ambisonic recordings.

1. Introduction

Time and frequency independent spatial transformations of Ambisonic recordings can be achieved by a simple matrix multiplication of the Ambisonic signals. Finding suitable transformation matrices for the post production of Ambisonic recordings is the goal of this article. Based on the "dominance effect" proposed by Gerzon [1] for adjusting the front-back balance Zotter and Pomberger [2][3] presented Warping of Higher Order Ambisonics for correcting the surround image. While deriving analytic expressions for the matrix coefficients can be challenging a straight forward numerical approach of finding the transformation matrix is mentioned in [3]. Ambisonic transformation matrices have also been studied by Chapman and Cotterell [4], however their article concludes with the erroneous speculation that the dominance transform only exists for fist order Ambisonics. By contrast, there are simple ways to describe any Ambisonic transformation.

This paper describes the simplest way to describe any Ambisonic transformation by performing all manipulation in the angular domain instead of the spherical harmonics domain. For a practical implementation, the Ambisonic signals are sampled at sufficiently many discrete points in the angular domain, where the location of the sampling points or their values are manipulated. Re-expansion of the manipulated angular samples back into the spherical harmonics domain yields the Ambisonic transformation matrix.

The presented manipulations are implemented in the `ambiX` audio plug-in suite [5] which can be downloaded at the authors website[1].

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2. Ambisonics

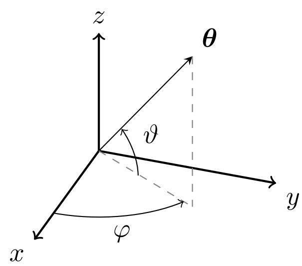


Figure 1: Cartesian and spherical coordinate system

We define our coordinate system as following (cf. Fig. 1): the x-axis points to the front, the y-axis to the left and the z-axis to the top of the listener. Within Ambisonics we mostly deal with spherical coordinates whereby φ is the azimuthal angle in mathematical positive orientation (counter-clockwise)[2] and ϑ being the elevation angle with $0°$ pointing to the equator and $+90°$ pointing to the north pole.

To denote the directional dependency of the surround signal represented by Ambisonics, we will often need to convert between a Cartesian unit direction vector

$$\boldsymbol{\theta} = \begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \end{pmatrix} = \begin{pmatrix} \cos\varphi\cos\vartheta \\ \sin\varphi\cos\vartheta \\ \sin\vartheta \end{pmatrix} \tag{1}$$

and the azimuth and elevation angles (φ, ϑ) of the spherical coordinates

$$\varphi = \arctan\frac{\theta_y}{\theta_x}, \qquad \vartheta = \arctan\frac{\theta_z}{\sqrt{\theta_x^2+\theta_y^2}}. \tag{2}$$

[2] The user interface of the software differs from this angle convention and uses a clockwise azimuth.

Ambisonics uses a spherical harmonic expansion up to order N to represent a surround audio signal $f(\varphi, \vartheta, t)$

F.Z., factor missing
Sept. 28th, 2016.

$$f(\varphi, \vartheta, t) = \sum_{n=0}^{\mathrm{N}} \sum_{m=-n}^{n} Y_n^m(\varphi, \vartheta)\ \phi_{nm}(t)(2n+1) \tag{3}$$

whereby Y_n^m being the spherical harmonics of order n, degree[3] m (Fig. 2) and $\phi_{nm}(t)$ the expansion coefficients.

Although not essential for the transformations in this paper it should be mentioned here that variations about the sequence and normalization of the spherical harmonics yield to difficulties in exchanging Ambisonic recordings or using Ambisonics software from different authors. Spherical harmonics consist of a normalization term $N_n^{|m|}$, the associated Legendre function $P_n^{|m|}$ and the trigonometric function,

$$Y_n^m(\varphi, \vartheta) = N_n^{|m|} P_n^{|m|}\left(\sin(\vartheta)\right) \begin{cases} \sin|m|\varphi, & \text{for } m < 0 \\ \cos|m|\varphi, & \text{for } m \geq 0. \end{cases} \tag{4}$$

The software associated with this publication complies to the ambiX [6] convention, therefore the Ambisonic Channel Numbering (ACN) Eq. (5) and SN3D normalization Eq. (6) are used. However the factor $\sqrt{\frac{1}{4\pi}}$ in the definition of $N_n^{|m|}$ from [6] resulting in a reduction of the signal by $\simeq 11$dB is neglected

$$ACN = n^2 + n + m, \tag{5}$$

$$N_n^{|m|} = \sqrt{(2 - \delta_m)\frac{(n - |m|)!}{(n + |m|)!}}. \tag{6}$$

Using this index neatly defines a sequence for the spherical harmonics $Y_n^m(\varphi, \vartheta) = Y_{ACN}(\varphi, \vartheta)$ and the Ambisonic signals $\phi_{ACN}(t)$ to stack them in a vector

$$\boldsymbol{y}(\boldsymbol{\theta}) = \begin{pmatrix} Y_0(\boldsymbol{\theta}) \\ \vdots \\ Y_{(\mathrm{N}+1)^2-1}(\boldsymbol{\theta}) \end{pmatrix}, \quad \boldsymbol{\phi}(t) = \begin{pmatrix} \phi_0(t) \\ \vdots \\ \phi_{(\mathrm{N}+1)^2-1}(t) \end{pmatrix}. \tag{7}$$

The angular dependency above is symbolically condensed by using the unit Cartesian direction vector $\boldsymbol{\theta}$ instead of (φ, ϑ). In the vector notation, surround signals as in Eq. (3) are conveniently written as

F.Z., Sept 28th, 2016:
factor for expansion missing.

$$f(\boldsymbol{\theta}, t) = \boldsymbol{y}^{\mathrm{T}}(\boldsymbol{\theta})\,\mathrm{diag}(a)\,\boldsymbol{\phi}(t). \tag{8}$$

with diag(*a*), with *a* = [(2n_ACN+1)/(4pi)],

Many publications use $\cos(\vartheta)$ as argument of $P_n^{|m|}$ which complies with the standard definition of the spherical coordinates, in which ϑ is not an elevation, but a zenith angle that is zero at the zenith and not at the horizon. This, however, was considered to be counter-intuitive in the musical application. Further [7] does not use the absolute value $|m|$ in the argument of the trigonometric functions yielding to a sign inversion of all Y_n^m with $m < 0$ which results in mirroring the y-axis[4] (see 2.1).

[3] some publications use the term order and degree in the opposite way

[4] this convention is also used in the Pure Data object `mtx_spherical_harmonics`

As there are different conventions in channel ordering, normalization, and sign inversion, the plug-in `ambix_converter` allows to convert between the different conventions on the fly.

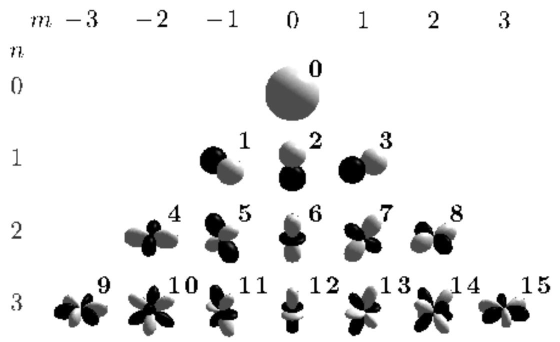


Figure 2: Spherical harmonics up to 3^{rd} order with Ambisonic Channel Numbering, order n and degree m

2.1. Transformations through symmetry considerations

For mirroring our Ambisonic signal $\boldsymbol{\phi}(t)$ about the coordinate system axes we can use the symmetry properties of the spherical harmonics [8]. These transformations can be applied with the `ambix_mirror` plug-in.

From the ACN index we get the order n with

$$n = floor(\sqrt{ACN}), \tag{9}$$

and the degree m with

$$m = ACN - n^2 - n. \tag{10}$$

We can mirror the y-axis (flip) by inverting all $\boldsymbol{\phi}(t)$ with $m < 0$ and therefore exchange the left and the right side of the surround image. The x-axis is mirrored (flop) by inverting all $\boldsymbol{\phi}(t)$ with $((m < 0)$ AND $(m$ even$)$ OR $(m \geq 0)$ AND $(m$ odd$))$ and exchange front and back. Mirroring the z-axis (flap) results in exchanging the top and bottom and can be realized by inverting all $\boldsymbol{\phi}(t)$ where $l + m$ is an odd number.

3. Transformations in the spatial domain

We are interested in finding the matrix $\boldsymbol{T}$ which expresses that the Ambisonic signals $\boldsymbol{\phi}(t)$ underwent spatial manipulation (e.g. mastering) to obtain modified Ambisonic signals $\tilde{\boldsymbol{\phi}}(t)$

$$\tilde{\boldsymbol{\phi}}(t) = \boldsymbol{T}\ \boldsymbol{\phi}(t). \tag{11}$$

To get such a matrix, we first consider relevant useful changes in the spatial domain. Desirable transformations of the surround signal are (1) weighting by a direction-dependent gain that helps to emphasize/attenuate signals form wanted/unwanted directions (2) angular transformations to modify the panorama of sounds in the surround signal. A surround signal $f(\boldsymbol{\theta}, t)$ in which each direction is weighted by $g(\boldsymbol{\theta})$ and mapped to another direction $\mathcal{T}\{\boldsymbol{\theta}\}$ in the new surround signal $\tilde{f}(\boldsymbol{\theta}, t)$ would be written as

$\tilde{f}(\mathcal{T}\{\boldsymbol{\theta}\},t) = g(\boldsymbol{\theta})\, f(\boldsymbol{\theta},t)$. An invertible directional mapping $\mathcal{T}^{-1}\{\cdot\}$ will simplify things later, therefore we use

$$\tilde{f}(\boldsymbol{\theta},t) = g(\boldsymbol{\theta})\, f(\mathcal{T}^{-1}\{\boldsymbol{\theta}\},t) \tag{12}$$

to search the corresponding matrix $\boldsymbol{T}$. To do so we insert Eq. (8) for $g(\boldsymbol{\theta}) f(\boldsymbol{\theta},t)$ and $\tilde{f}(\mathcal{T}\{\boldsymbol{\theta}\},t)$

diag(a) diag(a)

$$\boldsymbol{y}^{\mathrm{T}}(\boldsymbol{\theta})\, \tilde{\boldsymbol{\phi}}(t) = g(\boldsymbol{\theta})\, \boldsymbol{y}^{\mathrm{T}}(\mathcal{T}^{-1}\{\boldsymbol{\theta}\})\, \boldsymbol{\phi}(t). \tag{13}$$

To remove $\boldsymbol{y}^{\mathrm{T}}(\boldsymbol{\theta})$, we utilize the orthogonality of the spherical harmonics $\int_{\mathbb{S}^2} \boldsymbol{y}(\boldsymbol{\theta})\, \boldsymbol{y}^{\mathrm{T}}(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} = \mathrm{diag}\{\boldsymbol{a}\}$ to get:

diag(a)

$$\tilde{\boldsymbol{\phi}}(t) = \underbrace{\mathrm{diag}\{\boldsymbol{a}\}^{-1} \int_{\mathbb{S}^2} \boldsymbol{y}(\boldsymbol{\theta})\, g(\boldsymbol{\theta})\, \boldsymbol{y}^{\mathrm{T}}(\mathcal{T}^{-1}\{\boldsymbol{\theta}\})\, \mathrm{d}\boldsymbol{\theta}}_{:=\boldsymbol{T}}\, \boldsymbol{\phi}(t),$$

$$\text{with } \boldsymbol{a} = \left[\left(\tfrac{4\pi}{2n_{ACN}+1}\right)^{-1}\right]_{ACN}. \tag{14}$$

Numerical calculation of the hereby obtained transformation is found by expressing the integral over $\boldsymbol{\theta}$ in a discretized fashion. We recognize it as a spherical harmonics transform over the expression $\boldsymbol{T} = \mathcal{SHT}\{g(\boldsymbol{\theta})\, \boldsymbol{y}^{\mathrm{T}}(\mathcal{T}^{-1}\{\boldsymbol{\theta}\})\}$. diag($a$)

3.1. Discrete spherical harmonics transform

In order to implement the spherical harmonics transform of $g(\boldsymbol{\theta})\, \boldsymbol{y}^{\mathrm{T}}(\mathcal{T}^{-1}\{\boldsymbol{\theta}\})$, Eq. (14), the most convenient is to sample the transformation integral by a suitable distribution of, say L, directions

$$\boldsymbol{\Theta} = [\theta_1,\, \dots,\, \theta_{\mathrm{L}}]^T \tag{15}$$

on the sphere and to perform a discrete spherical harmonics transform $\boldsymbol{T} = \mathcal{DSHT}\{\mathrm{diag}\{\boldsymbol{g}(\boldsymbol{\Theta})\}\, \boldsymbol{Y}(\mathcal{T}^{-1}\{\boldsymbol{\Theta}\})\}$. diag($a$) A discussion of different sampling strategies, their aliasing characteristics and transform properties can be found in [7] and [9].

In general, $\mathcal{DSHT}$ requires to discretize the $(\mathrm{N}+1)^2$ spherical harmonics $\boldsymbol{y}(\boldsymbol{\theta})$ by the $\mathrm{L} \geq (\mathrm{N}+1)^2$ directions and to write the result into a $\mathrm{L} \times (\mathrm{N}+1)^2$ matrix, as well as discretization of $g(\boldsymbol{\theta})$ to the $\mathrm{L} \times 1$ vector $\boldsymbol{g}(\boldsymbol{\Theta})$

$$\boldsymbol{Y}(\boldsymbol{\Theta}) = [\boldsymbol{y}(\boldsymbol{\theta}_1),\, \dots,\, \boldsymbol{y}(\boldsymbol{\theta}_{\mathrm{L}})]^{\mathrm{T}}, \tag{16}$$

$$\boldsymbol{g}(\boldsymbol{\Theta}) = [g(\boldsymbol{\theta}_1),\, \dots,\, g(\boldsymbol{\theta}_{\mathrm{L}})]^{\mathrm{T}}. \tag{17}$$

The right-inverse $\boldsymbol{Y}^{\dagger} = (\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{Y})^{-1}\boldsymbol{Y}^{\mathrm{T}}$ achieves $\mathcal{DSHT}$, which yields in our application:

$$\begin{aligned} \boldsymbol{T} &= \mathcal{DSHT}\{\mathrm{diag}\{\boldsymbol{g}(\boldsymbol{\Theta})\}\, \boldsymbol{Y}(\mathcal{T}^{-1}\{\boldsymbol{\Theta}\})\} \quad \text{diag}(a) \qquad (18)\\ &= \boldsymbol{Y}^{\dagger}(\boldsymbol{\Theta})\, \mathrm{diag}\{\boldsymbol{g}(\boldsymbol{\Theta})\}\, \boldsymbol{Y}(\mathcal{T}^{-1}\{\boldsymbol{\Theta}\}). \quad \text{diag}(a) \end{aligned}$$

$\boldsymbol{T}$ is constant as long as the angular transformation $\mathcal{T}\{\boldsymbol{\Theta}\}$ and the weighting function $\boldsymbol{g}(\boldsymbol{\Theta})$ are not changing, which, however, might be necessary in applications with time-varying curves.

Note that angle-distorting or directional-loudness-weighting manipulation often requires $\tilde{\boldsymbol{\phi}}(t)$ to be of higher orders than the original Ambisonic signals $\boldsymbol{\phi}(t)$, which is not explicitly spelled out here, but can exemplarily be found in [2, Tab.1].

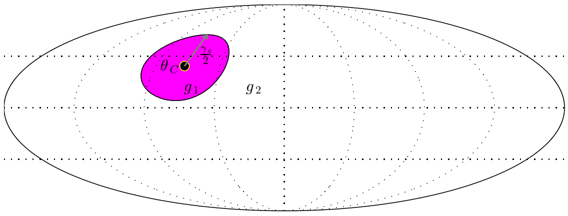


Figure 3: Spherical cap with center θ_C, size $\frac{\gamma_c}{2}$, gain factor g_1 inside the cap and g_2 outside the cap.

***t*-designs:** To achieve a low computational effort, a small number of sampling points would be beneficial. However, the number of sampling points must at least be $\mathrm{L} \geq (\mathrm{N}+1)^2$. Moreover, the condition number of $\boldsymbol{Y}$ needs to be sufficiently small to avoid numerical errors. The most pragmatic choice of sampling was presented by Hardin and Sloane [10], who provide coordinates $\boldsymbol{\Theta}_t$ for various spherical t-designs. For a transform of the Ambisonic order N, we would need a t-design of $t \geq 2\mathrm{N}$. The maximum currently available 21-design with 240 points allows the use up to Ambisonic order $\mathrm{N} = 10$, and it allows a $\mathcal{DSHT}$ without any pseudo-inversion:

$$\boldsymbol{T} = \mathrm{diag}\{\boldsymbol{b}\}\, \boldsymbol{Y}^{\mathrm{T}}(\boldsymbol{\Theta}_t)\, \mathrm{diag}\{\boldsymbol{g}(\boldsymbol{\Theta}_t)\}\, \boldsymbol{Y}(\mathcal{T}^{-1}\{\boldsymbol{\Theta}_t\}), \text{diag}(a)$$

$$\text{with } \boldsymbol{b} = \left[\left(\tfrac{2n_{ACN}+1}{\mathrm{L}}\right)^{-1}\right]_{ACN}. \tag{19}$$

Due to its practical advantage, all subsequent transformations use the above Eq. (19).

3.2. Directional loudness modifications

Modifying the loudness of specific directions is especially useful for post production of microphone array recording and is implemented in the plug-in `ambix_directional_loudness`. Svensson et al. [11] is using a modified set of basis functions to suppress signals from specified directions.

To performing loudness modifications in the angular domain, we consider a cap function to crop out a part of the surround sound scene (Fig. 3). For this purpose, we use Eq. (19) with neutral directional mapping $\mathcal{T}\{\boldsymbol{\theta}\} = \boldsymbol{\theta}$ and a gain function $g(\boldsymbol{\theta})$ that corresponds to a cap of unity amplitude of the size $\frac{\gamma_c}{2}$ around our center of the cap $\boldsymbol{\theta}_c$, and to zero elsewhere:

$$g(\boldsymbol{\theta}) = u(\boldsymbol{\theta}_c^{\mathrm{T}}\boldsymbol{\theta} - \cos\tfrac{\gamma_c}{2}), \tag{20}$$

with $u(\cdot)$ representing the unit step function. Other, more rounded functions will help keeping the order small, but will not be discussed here for simplicity. In practice, another gain function might be valuable that allows to determine gains for both the region within and outside the spherical cap with g_1 and g_2, respectively,

$$g(\boldsymbol{\theta}) = g_1\, u(\boldsymbol{\theta}_c^{\mathrm{T}}\boldsymbol{\theta} - \cos\tfrac{\gamma_c}{2}) + g_2\, u(\cos\tfrac{\gamma_c}{2} - \boldsymbol{\theta}_c^{\mathrm{T}}\boldsymbol{\theta}). \tag{21}$$

3.3. Rotation in three dimensions

Rotations of spherical harmonics around the z-axis are fairly easy to implement. More challenging is the rotation around the x- and y-axis which can be realized as combination of fixed 90° rotations around the y-axis and variable rotations around the z-axis [7]. To avoid the derivations in the spherical

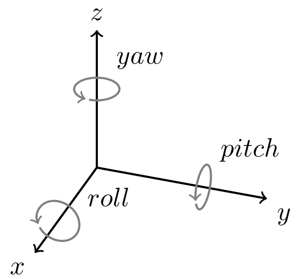


Figure 4: Rotation around x, y and z-axis.

harmonic domain we can perform the rotation in the angular domain. The rotation of the unit Cartesian direction vector $\boldsymbol{\theta}$ around the x axis (ϕ, roll), y axis (θ, pitch) and z axis (ψ, yaw) is done by multiplication with the rotation matrix $\mathbf{R}(\phi, \theta, \psi)$

$$\tilde{\boldsymbol{\theta}} = \mathcal{T}\{\boldsymbol{\theta}\} = \mathbf{R}(\phi, \theta, \psi)\, \boldsymbol{\theta}, \tag{22}$$

where

$$\mathbf{R}(\phi, \theta, \psi) = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}}_{\mathbf{x-axis-rotation(roll)}} \cdot \underbrace{\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}}_{y-axis-rotation(pitch)} \cdot \underbrace{\begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{z-axis-rotation(yaw)}. \tag{23}$$

The inverse transformation required to obtain $\boldsymbol{T}$ with Eq. (19) just uses the transposed matrix $\mathcal{T}^{-1}\{\cdot\} = \boldsymbol{R}^{\mathrm{T}}(\phi, \theta, \psi) \cdot \{\}$ and a neutral weight $g(\boldsymbol{\theta}) = 1$. Rotation does not increase the Ambisonic order.

3.4. Warping

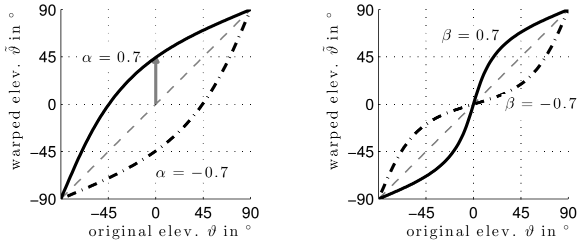
Warping is used to stretch a certain region of the surround image while squeezing it in other regions to prevent overlap. In this paper we describe the warping operation for the elevation angle ϑ to $\tilde{\vartheta}$. Warping along any other direction can be done by pre- and post-rotation. Zotter and Pomberger [3] mention the necessity for a magnitude emphasis as correction for the enlargement of sources after applying warping. Therefore, other than rotation and directional loudness manipulation, non of both modifiers $g(\boldsymbol{\theta})$ and $\mathcal{T}\{\cdot\}$ will be neutral in Eq. (19).

We are using a substitution to simplify subsequent warping curves, cf. [3], in order to express the manipulation of the angle ϑ in Eq. (2),

$$\begin{aligned} \mu &= \sin(\vartheta), && \text{original}, \\ \tilde{\mu} &= \sin(\tilde{\vartheta}), && \text{warped}, \end{aligned} \tag{24}$$

and we restrict ourselves to monotonically rising warping curves $\frac{\partial \tilde{\mu}}{\partial \mu} \geq 0$ that map μ of the interval $[-1, 1]$ to $\tilde{\mu}$ covering the interval $[-1, 1]$. To apply this manipulation on $\boldsymbol{\theta}$, the determination and modification of its angles φ and ϑ as defined in Eqs. (1) and (5) are required.

Warping to elevate or lower equator. As proposed by Gerzon and in [2], a bilinear transform provides a useful warping transformation between μ and $\tilde{\mu}$



(a) towards north pole and south pole, arrow indicates the shift of the equator $\arcsin\alpha$

(b) away from equator ($\beta > 0$) and towards equator ($\beta < 0$)

Figure 5: Warping of the elevation angle

$$\tilde{\mu} = \frac{\mu + \alpha}{1 + \alpha\mu}. \tag{25}$$

The operation is neutral for $\alpha = 0$, and depending on the sign of α, it elevates or lowers the equator $\vartheta = 0$ of the original surround image to $\tilde{\vartheta} = \arcsin\alpha$ for any α between $-1 \leq \alpha \leq 1$ (Fig. 5(a)). To preserve the total loudness of sounds within the stretched and squeezed parts of the surround sound images, the gain weight $g = \sqrt{\frac{\partial \tilde{\mu}}{\partial \mu}}$ needs to be applied after warping as *post-emphasis*, cf. [3],

$$g(\mu) = \frac{\sqrt{1 - \alpha^2}}{1 + \alpha\mu}. \tag{26}$$

Warping towards and away from equator. The following equation is another useful warping curve preserving the elevation of the equator. It is neutral for $\beta = 0$, pushes surround sound content away from the equator to the poles for $\beta > 0$, or pulls it towards the equator $\beta < 0$ (Fig. 5(b)),

$$\tilde{\mu} = \begin{cases} \frac{(|\beta|-1) + \sqrt{(|\beta|-1)^2 + 4|\beta|\mu^2}}{2|\beta|\mu}, & \text{for } \beta > 0, \\ \frac{(1-|\beta|)\mu}{1-|\beta|\mu^2}, & \text{for } \beta < 0. \end{cases} \tag{27}$$

The gain post-emphasis $g = \sqrt{\frac{\partial \tilde{\mu}}{\partial \mu}}$ is

$$g(\mu) = \left(\frac{1 - |\beta|\mu^2}{\sqrt{(1 - |\beta|)(1 + |\beta|\mu^2)}} \right)^{\mathrm{sgn}\{\beta\}}. \tag{28}$$

The exponent denotes that for negative β post-emphasis uses the reciprocal value of the expression in brackets.

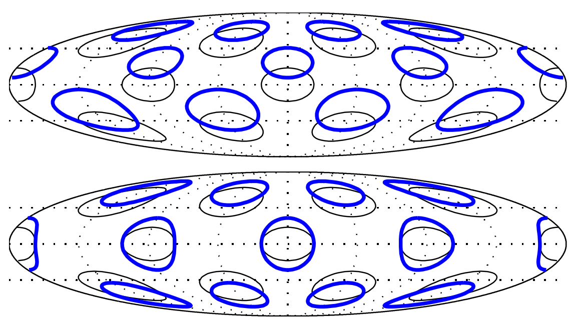


Figure 6: Warping scheme, thin lines indicates unmodified surround image, warping towards the northpole $\alpha = 0.4$ and warping away from equator $\beta = 0.4$

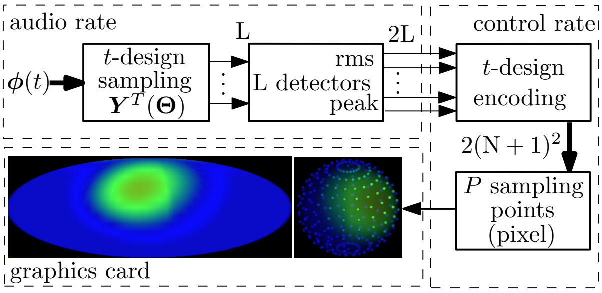


Figure 7: Ambisonic directional loudness meter

4. Metering Ambisonic signals

To support the verification process of the described Ambisonic manipulations a visual feedback of the directional loudness level is presented. The process of generating the visual surround image is divided in three parts, one operating in audio rate, one in control rate which defines the display frame rate and the final drawing part which is done on the graphics card (Fig. 7). The Ambisonic signal $\phi(t)$ is sampled with the t-design spherical harmonic matrix $\boldsymbol{Y}^T(\boldsymbol{\Theta})$ diag(a) resulting in L directional audio signals $\boldsymbol{f}(\boldsymbol{\Theta}, t)$. The rms and peak value of those L signals are measured with adjustable release time. Filtering may be applied before the rms/peak detectors to have a frequency selective display. The rms/peak detectors output their logarithmic measurements in the final display frame rate. These 2L values get re-encoded into the spherical harmonics domain with the same t-design used before. A texture with a resolution of v vertical and h horizontal pixel is used to display the directional rms value. The color of the $P = hv$ pixel is determined by sampling the rms spherical harmonic vector using a subdivision of h for the azimuth and v for the elevation. Afterwards this texture is mapped onto a sphere. The peak values are sampled in lower resolution and displayed as small spheres located as grid on the surface of the sphere. Additionally to the 3D view of the sphere the Mollweide projection is used to map the whole surround image on a 2D image.

The prototype visualisation has been developed using Pure Data. To reduce the CPU load it might be feasible to move all processing to the graphic card which is usually not busy with audio applications.

5. Conclusion

A pragmatic approach for calculating Ambisonic transformation matrices has been presented. These transformations can be used to attenuate or boost certain directions in Ambisonic recordings, to rotate, and to warp the spatial image in certain directions. The algorithms have been implemented as ready-to-use audio plug-ins applicable to production and postproduction of Ambisonic recordings. Additionally the transformations can be used to adapt Ambisonic recordings to certain playback situations. For all that, a new way of metering the Ambisonic surround production is required and was successfully presented.

6. Acknowledgments

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F.Z., Sept. 28th, 2016: Thanks to Alexander Krüger (Sennheiser) for pointing out the missing normalization factor (2n+1) which was not noticed our code operating on N3D, internally.

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